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THE ART OF LAND-MEASURING EXPLAINED.

IN FIVE PARTS.

VIZ.

I. TAKING DIMENSIONS.	IV. DIVIDING.
II. FINDING CONTENTS.	AND
III. LAYING OUT GROUND.	V. PLANNING.

WITH AN APPENDIX CONCERNING INSTRUMENTS.

BY JOHN GRAY,
TEACHER OF MATHEMATICS IN GREENOCK,
AND LAND-MEASURER.

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TO THE

RIGHT HONOURABLE,

CHARLES SCHAW

OF SAUCHIE,

LORD CATHCART.

MY LORD,

THOUGH I cannot, like most other Authors, claim the honour of acquaintance, and favours received, Your LORDSHIP's known readiness to encourage every attempt to cultivate and improve the useful arts, and that particular regard with which every thing relating to this town of GREENOCK has always been kindly distinguished by Your LORDSHIP, em-

bolden me to aſpire to the honour of having the name of My LORD CATHCART prefixed to this Book, and of ſubſcribing myſelf in this public manner,

MY LORD,

YOUR LORDSHIP'S

MOST OBEDIENT, AND

MOST HUMBLE SERVANT,

JOHN GRAY.

THE PREFACE.

THE title page, and the following table of contents, will ſhow what may be expected in this book. I ſuppoſe a long preface needleſs, and only beg leave to obſerve, that it is not ſurveying of kingdoms, or very large tracts of land, that I here propoſe to explain; but ſuch ſurveys as are intended, principally, for finding the contents of the ground ſurveyed, or for the contents and plan both, with as much exactneſs as poſſible.

As for my differing from former authors in the general method, and ſome particular things, ſuch as condemning inſtruments of a ſmall radius &c. and propoſing others in their ſtead, which I think are better; I humbly hope, a good intention, and honeſt regard for truth, will plead in my behalf. Why ſhould long cuſtom, or any other prejudice, hinder any from uſing better inſtru-

ments, and more exact methods, if they can be had? As this art, like all others, as far as I know, is not yet brought to perfection, some improvements may be attempted. How far I have succeeded is not for me to determine.

If what I have here published be acceptable to the public, and prove one cause of producing something, by an abler hand, that shall deserve the thanks of all concerned with land-measuring, I have my desire.

THE CONTENTS.

PART I.

PART II.

PART III.

PART IV.

PART V.

THE INTRODUCTION.

MANY are the books already publiſhed upon the ſubject of ſurveying or meaſuring of land. Many and long are the rules given for the ſeveral purpoſes of taking the dimenſions and plotting. Great is the apparatus of inſtruments for the ſingle purpoſe of meaſuring angles. Yet I humbly think the ſubject is ſo far from being exhauſted, that, as far as I know, there has not appeared one, that has explained the art ſo fully and clearly as might reaſonably be expected from the evidence of its principles, the uſefulneſs, frequency, and eaſineſs of the practice. This laſt particular has, perhaps, made it appear below the notice of ſeveral men of learning and genius, who are the fitteſt to explain any ſubject, but who for the moſt part ſpend their time and pains upon ſuch only, as (they think) require theſe qualifications, by reaſon of ſome difficulty either in the theory or the practice, or in both: but ſurely there can be no reaſon for thoſe who have written profeſſedly

to explain this ſubject, not to give all the rules, which they know to be obſerved by good ſurveyors, and to inſiſt largely upon ſuch as, they know, cannot be applied with tolerable exactneſs in practice. Our authors, certainly, have not known theſe things: and the art has been improved ſince they wrote, or were acquainted with the practice.

My deſign in the following tract is, to attempt as full and clear an explication, as I can give, of the whole art of land-meaſuring as it is now practiſed in this country: and under this title I comprehend, I. Taking the dimenſions of any piece of ground that is meaſurable; that is, where all the lines, by which the content can be found, may be meaſured, or calculated by ſuch lines and angles as are neceſſary to find thoſe and may be meaſured upon the ground. II. Making a plan or map of the ground meaſured. III. Finding the content of ground of any figure, by the lines meaſured and calculated. IV. Laying out any content propoſed in any figure that may be required. And, V. Dividing ground into any parts required.

The deſign of this introduction is, to make ſome general obſervations naturally ariſing from a ſlight conſideration of each of theſe ſeveral heads; all drawn from experience, and confirmed by a great variety of practice for the ſpace of many years. By experience I mean,

both my own and that of others, which I have had access to know: and by variety of practice, all the different kinds of surveys, that can well be expected, from a cottage yard or single rig, to a whole parish or county. To begin then with the first head.

I. Taking dimensions &c. Here it will not be improper to consider a little the instruments used for this purpose. They are, the Chain, Rod, Wheel &c. for Lines: the Quadrant, Semicircle, Theodolite, Circumferentor, Cross-staff, Peractor, Imperial table &c. &c. &c. for angles: all of them except the cross-staff, with a radius under 6 inches: nay some of them, by way of improvement, under 3 inches. The chain is, I believe, universally allowed to be the best for measuring lines; but what if it should be found the best too, of all the above-named, for measuring angles, when it can be used for that purpose, and that is, wherever the ground is open and nearly level? This is what none of our authors have explained with respect to all kinds of angles; they have only shown how to measure an angle upon the surface of the ground by it, and that too, imperfectly: but how to take an angle of elevation by it, they have not so much as hinted, as far as I have seen or heard: yet this is often necessary, when quadrants &c. are not at hand, and can be done more exactly by it, than by any one of all the above instruments, and the rest of the same or a less radius, that are

not mentioned. Let this aſſertion be put to the trial. See Prob. X. of the firſt part following.

Suppoſe the diſtance from pin to pole equal to the difference of their heights, and conſequently the angle of elevation $45^\circ\ 00'$; which ſuppoſition bears hardeſt, upon this rule, of any that can be made; for the more the angle is increaſed above, or diminiſhed below $45^\circ\ 00'$, the leſs is the effect of an error in that diſtance or difference of the heights. If, inſtead of 10 links (for example) the diſtance is 9,98, which is an error of $\frac{1}{50}$ of a link, this will make the angle $45^\circ\ 03'$; and I can ſee nothing to hinder the meaſuring of 10 links to $\frac{1}{50}$ of a link of the truth, or to $\frac{1}{6}$ of an inch. Now how exactly can the common quadrant, theodolite &c. with a radius of 6 inches take any angle? let us ſee. The arch of the quadrant is 9,4 inches, a degree about $\frac{1}{10}$ of an inch and 6 minutes $\frac{1}{100}$. This $\frac{1}{100}$ of an inch is to be cut off by the exact half of a thread, hung from a centre, perhaps not that of the quadrant; for it is very poſſible to miſtake there, with a plummet, and hand ſhaking a little——I only ask, whether 6 minutes can be as ſurely taken this way, as 3 minutes may be the other way? and if this is the moſt favourable ſuppoſition for the quadrant——As the diſtance of the pin from the pole increaſes, the angle of elevation decreaſes, and an error in the diſtance may more eaſily eſcape. Let us then

make another ſuppoſition, of a diſtance of 25 links, and difference as before 10 links; the angle will be found now 21° 48′; and ſuppoſing the true diſtance 24,9, or an error of $\frac{1}{10}$ of a link, about $\frac{8}{10}$ of an inch in a length of 16$\frac{1}{2}$ feet; the true angle will be 21° 53′, ſtill more exact than the quadrant, which can hardly be depended on within 10 minutes, as may be concluded from what has been juſt now obſerved, and is really confirmed by experience.

As for the theodolite, and the other inſtrtıments above-mentioned, whoſe principal uſe is to meaſure an angle upon the ground, and which, particularly the theodolite, are ſo highly extolled; if the chain, with its neceſſary attendants, poles and pins, can take any angle upon the ground, where it can be applied as exactly as an angle of elevation, it muſt certainly be preferred to them alſo: let us ſee then if it can. See Prob. XVII. of Part I.

The chord may be meaſured exactly, at leaſt to $\frac{1}{10}$ of a link: the greateſt error that can happen is when the chord is near 100 links, for the ſhorter it is, the leſs is the effect of an error upon the angle: in this caſe it comes within 4 minutes of the truth, and in every other caſe nearer; whereas by the theodolite &c. as appears above, the neareſt you can come, is to 10 minutes.

If a chain then, which muſt be had at any rate, can do the buſineſs of all theſe inſtruments, on open level ground, tho' not ſo quickly, with far greater exactneſs, I would fain know what is the uſe of them there? and if ſuch an inſtrument, as the Graphometer, after deſcribed, as quick as any of theſe, that can be uſed on every ground, more convenient and more exact than the chain, can be eaſily got, may it not be preferred to any pretty gewgaw, of no real uſe at all?

But as it is the opinion of ſeveral authors and practitioners, that an angle taken within 10 minutes of the truth is exact enough, and accordingly you will find tables of logarithms calculated to every 5th minute only, publiſhed along with books of ſurveying; let us try what grounds there are for that opinion, and what the followers of it mean by exact enough.

Suppoſe then an incloſure of the figure of a rhomboides, its baſe is 1000 links, its angle 16° 20′, and ſide 1042, the area will be found 293050; but if the angle ſhould be 16° 10′, that would make the area 289900. Here the difference is 5 poles, in a content leſs than 3 acres, equal nearly to 1 rood 10 poles in about 29 acres: and this error affects the area without altering the ſide one link.

Suppoſe again, a triangle whoſe baſe is 1000 links, and angle oppoſite to the altitude 74° 15′, the area will

be found 17 acres, 2 roods, 36,6 poles: but ſhould the angle be 74° 25′, then the area would be 17 acres, 3 roods, 28,48 poles, the difference near 32 poles in leſs than 18 acres.

In both theſe examples, the error may be 1 acre in 88 or 90; the very leaſt error in any caſe will be found 1 in 173. Is this exact enough?——I don't ſay that any graduated inſtrument can be ſo very exact, in every angle, as to $\frac{1}{1000}$ of any altitude, ſuppoſing the angle taken within 1 minute of the truth; but ſurely, within 1 minute is better than only within 10; and an error of 1 in 900 is more excuſeable than 1 in 90: or upon the moſt favourable ſuppoſition, within 1 of 1720, is more exact than within 1 of 172.

But to ſhow that no advantage is taken by chuſing angles, let us make one ſuppoſition more; of an angle oppoſite to the altitude of a rhomboides, taken for 87° 50′, inſtead of 88° 00′; let the ſegment of the baſe next the angle be 20 links, and the whole baſe 1000, the altitude will now be found 528,625, inſtead of 572,725, and the difference of the areas above 1 rood and 30 poles in leſs than 6 acres.

In this caſe, of finding the altitude by the angle and ſegment of the baſe, or whole baſe, by which theſe three examples are wrought, you may obſerve, that the nearer the angle approaches to 90°, or the ſmaller

it is, the greater is the effect of an error: in the last example, the difference of 1 minute would make $4\frac{3}{4}$ in the altitude, and $7\frac{1}{2}$ poles in the area. There is only one other possible case, by which the altitude can be found, viz. by the angle and two including sides, or hypothenuse and base; and here an error of 1 minute under 15° has the same effect as in the other case; and it is the same thing above 165°; so that here are 30 degrees, or one sixth part of the number of all possible angles, which, if you please, you may call Forbidden Angles in both possible cases; and as many more, viz. within 15° of 90°, forbidden in one of the two cases by which the altitude and consequently the area can be found; for an error of a single minute has a considerable effect.

When these angles, therefore, occur in practice, as they must very frequently, the right lines subtending them must be measured, as well as they, and used instead of them: and where this cannot be done, that ground may be declared, not exactly measurable, by any method ye discovered. I don't mean perfect exactness; that is a thing not attainable: but that the error may not exceed 1 in 900.

If it should be objected, that this is too much of exactness to insist upon, because there are so many small errors almost unavoidable in measuring the lines, that not

content can be expected within 1 acre of 900. I answer, so much the worse for the theodolite &c. unless you can suppose their errors to balance some of the others; for if the sum of all these unavoidable errors, supposing the worst, should amount to 1 in 800 acres, sure, 1 more in 900 is enough in all conscience. But the exactness of measuring the lines may be tried too. The chain may be made exact to $\frac{1}{10}$ of an inch at least; suppose then an error of $\frac{1}{8000}$ in the make of the chain: any line whatever may be measured or found to 1 link in 10 chains, this will make the unavoidable error not above $\frac{1}{1000}$ in the measuring: the sum of these errors then is $\frac{9}{8000}$, or 1 in 888, at most. Nay, it may be very justly reckoned within 1 of 1000.

In the foregoing examples I have chosen the rhomboides and triangle, only for the easier proof: but the effect of the errors will be found the same upon the altitudes and areas of all other figures; and the greater the number of sides, the worse.

From the whole then, I think I may fairly enough infer, that a good graduated instrument may be trusted for all, except the forbidden angles; but the common theodolite &c. &c. &c. for no angle whatsoever; and that it is much better trusting the measures of lines than of angles, in all the cases that may occur in practice, because the exactness is greater, and always the same.

I ſhall conclude upon this head with obſerving, that every thing required to be done in the field, can be performed by the chain only, and without it, nothing. And the leſs you make the radius of a graduated inſtrument, the more uſeleſs it is.

II. Planning &c. This muſt be performed with a ſcale and compaſſes: and when there are no angles to be laid down, there is no need of any other inſtrument: but when angles are taken, ſome graduated inſtrument muſt be uſed in forming the plan. The protractor is that commonly recommended and uſed. Let us conſider it. Its radius is, at moſt, one half of that of the theodolite: you may almoſt diſtinguiſh $\frac{1}{3}$ of a degree from $\frac{2}{3}$. Let us ſee now, how truly a triangle, right-angled for example, may be planned by it. Suppoſe the baſe 2000 links, and the oppoſite angle 18° 30′, the other leg will be found 5977. But if the angle ſhould be 18° 10′, the leg would be 5864. Here, for any thing you know, may be an error in every 53 links of the ſide determined by protraction. If every triangle is wrong laid down, and if there are 40 triangles in the plan; what will be the conſequence? And yet a protractor has coſt ſome pounds, a theodolite a great many more: 'tis a pity ſuch dear companions ever ſhould be ſeparated; let them therefore live and die together!

What! no more theodolite? no more protractor?

No. I have propoſed a ſucceſſor to the firſt: and if you'll try, in place of the other, a ſector with lines of chords to a radius of 8 or 9 inches, it will do better; and if the radius be one foot, it will be ſo much better ſtill.

III. Calculation &c. Here, I cannot help thinking it moſt ſurprizing, that two things not having the leaſt degree of connection, planning a piece of ground, and finding its content, ſhould be made inſeparable! yet this is done by all the authors upon the ſubject that I have ſeen or heard of. They all direct to protract the figure of the ground which you have meaſured; then to meaſure the baſes and altitudes &c. upon this figure, except ſuch as were meaſured in the field; then to find the contents by theſe meaſures: nay, it is affirmed expreſly by ſeveral of them, that the content of no figure, except the rectangle, can be found without protraction. Strange! is there no other way to come near the truth, but by wading thro' falſhoods, and heaping errors upon errors? To the ſmall errors, almoſt unavoidable in taking the dimenſions by the chain, muſt we add, not only the errors, alſo unavoidable, in our ſcales and uſing of the compaſſes; for no inſtrument is perfect, nor can be perfectly well uſed, but alſo the certain and great errors of the theodolite and protractor? The dimenſions neceſſary for planning are ſufficient alſo for finding the content. The only things to be expected from the moſt exact

plan, are a figure of the ground pretty near the truth, and when laid down from a large ſcale, a gueſs at the content, made with a great deal of needleſs trouble: a Plain table draught indeed ſhould be excepted, from a ſcale of 200 links in an inch, but this is never called protraction, and is really a very different thing. We may come very near the truth by methods almoſt as eaſy. But what ſhall we ſay when we are not allowed? When ſurveying by the theodolite, whether there be a neceſſity of taking angles, or not, and planning that ſurvey by the protractor, is inſiſted upon, as the only way to find the true content? Ha!———Let us proceed to the next head.

IV. Laying out &c. This being very often required, and eaſily performed, one might reaſonably expect a clear and full explanation of all the rules for the purpoſe, from moſt of our authors: yet none, that I know of, have given more than two or three rules, by the by, for doing it: and theſe always ſuppoſe, that the baſe, upon which you are to lay out, is a right line without off-ſets, or irregular turnings and windings, upon any ſide of it: which is rarely the caſe; for in laying out new grounds, or in cutting off a piece from ground already brought in, we are commonly confined to a boundary on one ſide, and ſometimes more; which boundary is very often a crooked winding line, and muſt be our

baſe, in ſome part of the work. Now if all the ſmall turnings are overlooked, the content cannot be laid out truly, neither can it be known how far it is wrong, and conſequently the error is irremediable.

V. Dividing &c. Here again our authors all agree in the ſame neglect of rules ſufficient for the variety of the caſes, and in the ſame erroneous ſuppoſition as to the baſes and boundaries: but they do more: they direct you firſt to plan the ground (by the protractor) then to reduce this exact figure into a triangle, by drawing lines and arches; without telling how that is to be done upon the ground, or how you are to proceed, when it cannot be done there at all: then to divide &c. A method curious, artificial, wonderful, and in one (ſeſquipedalian) word, geometrically-ungeometrical!

To theſe obſervations upon each of the particular heads, I ſhall add one more regarding them all.

Unleſs the irregular turnings and ſmall windings, containing the off-ſets, that almoſt every where appear upon the boundaries of open fields, and very often in incloſures, are exactly meaſured; you can neither find the true content, lay out, divide, nor plan truly. This ſhould appear very evident: yet where is the book hitherto publiſhed, that gives any rule at all for this purpoſe?

If any author has yet appeared, who has obviated

all my objections, and taught in the exact way which I propoſe to attempt, I ſhall be well pleaſed not to be thought ſingular. But if all my objections, (for the foregoing obſervations are no leſs) or even ſome of them only, ſtrike againſt all the books yet publiſhed, the preſent ſtate of the art of land-meaſuring, as it appears in print, is not arrived at that perfection it is capable of. Neither do I think it will, by all that I can do for it. But I think, nothing ſhort of the greateſt exactneſs, that poſſibly can be attained to, ſhould ſatisfy any man, who teaches or practiſes any uſeful art.

LAND-MEASURING

EXPLAINED.

I SUPPOSE the learner of this art to have been taught arithmetic, vulgar, decimal and logarithmical; Euclid's Elements of Plain Geometry; and Plain Trigonometry: a competent knowledge of theſe is all that is neceſſary to prepare for it.

I ſhall divide the ſubject into five parts, treat of them in the following order, and make Planning the laſt, becauſe it has not the leaſt connection with any of them, excepting the firſt part only; and the other three parts are cloſely connected with the firſt and with one another. Theſe parts then ſhall be as follow, viz.

I. Surveying, or taking dimenſions.

II. Calculation, or finding contents.

III. Laying out ground.

IV. Diviſion of ground.

V. Planning, or making a map.

Thro' the whole, I ſhall uſe the form of Problem, Rule and Example, wherever neceſſary for explaining and connecting the ſeveral parts of the ſubject: proceeding from the more general and eaſy to the more particular and difficult.

PART I.

SURVEYING.

UNDER this title I propoſe to give all the neceſſary rules, for taking the dimenſions of any piece of ground, in order to find its content, lay out, divide or plan it. Theſe dimenſions are lines and angles. For meaſuring or making of which, you may uſe the following inſtruments, viz.

I. The Chain with poles and pins. II. Off-ſet ſtaves. III. The Semicircle or Graphometer. And, IV. The Quadrant. Beſides theſe a great many others have been and ſtill are uſed.

I. The Chain, invented by Mr. Edmund Gunter, about 150 years ago, is in length 22 yards, divided into 100 equal links, each link (conſiſting of a piece of iron or braſs wire with a ring at each end) is 7,92 inches: every 10 links to 50 may be marked with a ſmall plate of braſs (or iron) of a different form, the 50th having a mark different from all the reſt, 40 and 60 the ſame mark, ſo 30 and 70, 20 and 80, and 10 and 90. Each end may have a ſpike to ſtick into the ground, and make a hole for a pin to go into. You may likewiſe have a piece of wood or braſs, one link long, and

divided into 100 equal parts, to uſe when very great exactneſs is required, which, if you pleaſe, you may call a Decimal Link. In imitation of this chain, we have got a Scotch one, in length 24 ells, each link 8,88 Scotch inches. The poles may be 8 or 9 feet long, and ſhod with iron at the end which goes into the ground. The pins may be about 18 inches.

II. Off-ſet ſtaves ſhould be 10 links in length, and divided accordingly, with a croſs-head fixed perpendicularly. They ſhould be two, divided and marked the ſame way on both ſides. Their uſe is, to ſet off or meaſure a ſhort perpendicular in the field, which is done by placing one croſs-head on the line, and one ſtaff at the other's end, with the heads parallel, lifting the firſt to the end of the ſecond &c.

III. The Semicircle, or Graphometer, may be made of wood, or braſs: it is divided, by right lines drawn from the centre, into 180° 00′, and theſe degrees into minutes, if the circumference will allow, which it will eaſily, if the diameter be 3 feet, or above, and the degrees divided by diagonal lines, as the plain ſcales. Or the dividing lines may be produced from the limb of the ſemicircle, for every 10 minutes, upon the ſides of a rectangle without it, and the diagonals drawn there. It may have two fixed ſights upon the diameter, and a moveable index on the centre with two ſights upon it.

It may be mounted upon the head of a three-legged ſtaff, which has a ball moving in a ſocket, to give it any poſition, level or ſloping any way required, which when you have got, you ſcrew the ball faſt, that the inſtrument may keep that poſition, till you have occaſion to give it any other. The ſtaff has three legs that ſpread wider and wider, as you would have the inſtrument elevated more or leſs. Its uſe is to meaſure angles in the field.

Note, An arch of $1^\circ\ 00'$ to a radius of 3 feet, divided as above, and fixed to a grove ſliding round the limb, may ſerve for all the degrees.

IV. The Quadrant is the 4th part of a circle, of wood or braſs, divided into degrees &c. If you make it like a bow, that is, having only the arch and chord, you may have a radius of 36 inches fixed into the middle of the chord and arch, or moveable out and in, for eaſe and ſafety in carrying, and the arch may be divided as before. It may have an index of braſs, or iron, hanging down from the centre upon the arch, and the whole as light and more portable than a common quadrant of 18 inches radius. It may be mounted the ſame way as the graphometer, and is for the ſame uſe; tho' not ſo quick, it is much more exact than the other, upon level ground, for an angle under 90°.

I ſhall now proceed to explain the practical rules for ſurveying in the following Problems, wherein I ſhall

give the Rules all diſtinct from the Examples, and unmixed with them, inſerting no more figures than are neceſſary to explain them. I ſhall do the ſame through the whole book.

PROBLEM I.

To meaſure a ſtraight line all viſible and acceſſible from each end; by the chain.

RULE.

Set up a pole at each of the ends, if there is no conſpicuous mark there already; let the foremoſt chain-bearer take 10 pins with him, and while the hindmoſt holds his end of the chain cloſe at the foot of the firſt pole or mark, let the foremoſt ſtretch out the chain its full length upon the ground toward the ſecond pole: look over the chain all along, cauſe the foremoſt man's end cover the ſecond pole from the hindmoſt, and there let the firſt pin be ſtuck down: let them go on toward the ſecond pole another chain's length; when the hindmoſt comes to the firſt pin, let him there hold faſt, direct the foremoſt as before, and when he has put down the ſecond pin, let the hindmoſt take up the firſt. Let them proceed in this manner, till the foremoſt comes to the pole, or till the hindmoſt has got up all the 10 pins; then let them go on another chain's length, and when the foremoſt has ſet his end as before, let him get all the pins from the hindmoſt, and ſtick down one.

In this manner proceed to the end pole. Be ſure to keep an exact account of the number of changes: for this purpoſe it will not be amiſs for the hindmoſt to put a ſmall ſtone in his pocket at every change; or you may cut a ſtroke upon a ſmall ſtick: it will not be amiſs too if you count the pins. When the foremoſt comes to the pole, count the pins the hindmoſt has got and the odd links to the pole. The number of changes, pins and odd links is the meaſure of the line required.

Thus if you have 9 changes, 8 pins, and 76 links, the whole line is 9876 links, or 98 chains, and 76 links.

Note, When the hindmoſt man loſes ſight of the end pole, the foremoſt may direct himſelf, by covering the firſt pole or laſt pin with that which he himſelf ſticks down. This may be alſo done, to make ſure of the ſtraightneſs of the line, tho' the pole be viſible.

PROB. II.

To meaſure a ſtraight line, part of which goes thro' a hollow, the reſt viſible from both ends, and all of it acceſſible: the hollow not very deep, nor ſides ſteep; by the chain.

RULE.

Meaſure as before to the hollow; there cauſe a pole to be ſet up, or obſerve a mark on the other ſide in a right line with both the end poles, or covering the one

from the other: meaſure to this pole or mark; and from it to the other end, as before.

PROB. III.

To meaſure a ſtraight line, all acceſſible, but part of it not viſible from both ends, by reaſon of a ſmall riſing in the ground; by the chain.

RULE.

Having ſet the end poles, go to the riſing ground with one aſſiſtant, each of you carrying a pole; there, at ſome diſtance from each other, move with your poles perpendicular on the ground, till his hides one end pole from you and yours, the other from him: there fix one of your poles. Meaſure from the firſt pole to this laſt ſet pole, and from it to the end, as before.

PROB. IV.

From a given point in a given right line, to raiſe a perpendicular; by the chain.

RULE I.

Fix the end of the 24th link on the point, ſtretching 24 upon the line: let the firſt end, and the end of the 96th be kept faſt upon the line: take the end of the 56th in your hand, and ſtretch the chain, as it now lies on the ground, to the full extent: ſtick down a pin at the end of the 56th, and a pole at ſome diſtance in a right line with the pin and point. Produce this line from the point as far as neceſſary, it is the perpendicular

required. For here is a triangle having one ſide 24, another 32, and the third 40, being in the ratio of 3, 4, 5. And all ſuch triangles are right-angled.

RULE II.

Fix the two ends of the chain upon the line 30 or 40 links diſtant from the point. Take the end of the 50th in your hand, and ſo ſtretch to the full extent: then ſtick a pin &c. as before.

Note, A ſhort perpendicular may be raiſed by the eye, or the chain may be directed by the off-ſet ſtaves, as in page 17th.

PROB. V.

From a given point of a right line, to raiſe a perpendicular, by graphometer, or quadrant.

RULE.

Set the centre above the point and the diameter, or radius to 00° 00′, upon the line, and there ſcrew faſt: turn the index upon 90° 00′, and keeping it there, cauſe a pole to be ſet in a right line with it: run a line from the point upon this pole: it is the perpendicular required. If you uſe the bow quadrant, the index is meant by the radius, or a right line from the centre to 00° 00′.

PROB. VI.

To find a right line thro' a wood, marſh &c. one end of which you can paſs by; by the chain or graphometer.

RULE.

Meaſure to the ſide of it, as before: there raiſe a perpendicular to reach to the end which you can paſs by: from the top of this, raiſe another reaching to the other ſide: from the top of this a third, equal to the firſt perpendicular: then from the top of this third, raiſe a fourth to the end of the line. Add the firſt meaſured part, the ſecond and fourth perpendiculars, the ſum is the length of the whole line. See Plate I. Fig. 1.

Thus the ſum of AC + DE + FB = AB. Here you may obſerve that the line DE is parallel to the line AB. See another method in Prob. XXI.

PROB. VII.

To let fall a perpendicular from a given point upon a given right line; by a graphometer or quadrant.

RULE.

Set the index to 90° 00′, and keeping it there, with the diameter, or radius to 00° 00′ upon the line, move the inſtrument forward or backward, till the index be in a right line with the point: ſet a pole in the place immediately below the centre: take away the inſtrument, and run a right line from the point to the pole. It is the perpendicular required.

PROB. VIII.

To do the ſame by the chain, without any graduated inſtrument.

RULE.

Take the end of the 24th link in your hand, let one aſſiſtant take the end of the 96th and beginning of the 1ſt in his hand, while another holds the end of the 56th; carry the 24 links along upon the line, till the ſecond aſſiſtant brings the end of the 56th link into a right line with the point and your end of the 24th; the chain being ſtretched to the full extent: ſet a pole in place of the end of the 56th. Draw a right line from the point to the pole, and produce it to the line.

Note, A ſhort one may be let fall by the off-ſet ſtaves, turning the heads toward the line &c.

PROB. IX.

To take an angle of elevation or depreſſion, by the quadrant.

RULE.

At the foot of the height, direct the quadrant to the top, the index hanging down freely, and juſt touching the limb; it will ſhow you the meaſure of the angle. But for the angle of depreſſion; upon the top of the height, direct the quadrant with its centre to your eye, to the foot, and you will ſee the meaſure of the angle cut off upon the limb from you.

PROB. X.

To take an angle of elevation, by the chain.

RULE.

At the foot of the height, set a pole truly perpendicular; go back from it in an horizontal line, till you just see the top of a pin sticking in the ground, the top of the pole and the top of the height, all three in one line: measure the height of pole and pin above the ground, and distance, most exactly: then, as the exact distance of pole and pin, to the difference of their heights, so is radius, to the tangent of the angle of elevation.

EXAMPLE.

Suppose the height of the pole above the ground 8 feet 3 inches, that of the pin 1 foot 9 inches, and their distance 27,8 links. I demand the angle of elevation?

		F.	I.
Height	of the pole	8 --	3
	of the pin	1 --	9
Difference - -		6 --	6

L. 12

Distance 27,8 7,92)78,00(9,85 Links.
7,92 71 28
556 6720
2502 6336
1946 384
12)220,176
18 F. 4,176 Inches.

As the distance of pin and pole 27,8	Log.	1.44404
To the difference of their heights 9,85	L.	0.99344
So is radius, tangent of 45° 00′ -	L.	10.
To the tangent of the angle required 19° 31′ - - - -	L.	9.54940

Answer 19° 31′.

Note, You cannot take the angle of depression by the chain, but you can measure, or find the down-hill line, and at the foot take it as an angle of elevation. The height of pole and pin may be measured in links, and these reduced to feet &c. or E contra, as you see done above.

PROB. XI.

To find the bottom line thro' a hill; or to reduce hypothenusal lines to horizontal.

RULE.

Take the angle of elevation, and measure up to the top: then say, as radius to the up-hill line, so is the cosine of the angle of elevation to the bottom line that reaches to the foot of the perpendicular height. Take the angle of depression, and measure the down-hill line; then say, as radius to the down-hill line, so is the sine of the angle of depression to the rest of the bottom line. If the whole bottom line be required at once; say, as the sine of the angle of elevation to the down-hill line,

so is the sine of the sum of the angles of depression on both sides to the whole horizontal line.

EXAMPLE I.

At the foot of a hill, I find the angle of elevation 19° 10′, and measure up to the top 3480 links; I demand the length of the horizontal line to the foot of the altitude?

As radius, sine of 90° 00′	-	Log.	10.
To the up-hill line 3480	- -	L.	3.541579
So is the cosine of 19° 10′	-	L.	9.975233
To the horizontal line 3287	-	L.	3.516812

Answer 3287 links.

EXAMPLE II.

At the top of a hill, I take the angle of depression 75° 48′, and measure down to the foot 2865 links. I demand the horizontal line from the foot of the altitude to the foot of the hill?

As radius, sine of 90° 00′	-	Log.	10.
To the down-hill line 2865	-	L.	3.45712
So is the sine of 75° 48′	- -	L.	9.98652
To the bottom line 2777 links	-	L.	3.44364

Answer 2777 links.

EXAMPLE III.

For the whole bottom line of another hill. See the work.

Suppose angles of depression		{ 70° 50′
		{ 75 48
Of elevation 19° 10′	Sum	146 38
And down-hill line 2865		180 00
	Supplement	33 22

As sine of 19° 10′ - -	Log.	9.51629
To down-hill line 2865 -	L.	3.45712
So is sine of 146° 38′ - -	L.	9.74036
		13.19748
To the bottom line 4799 -	L.	3.68119

As the horizontal lines are the true lines, either for finding the content or forming the plan of the ground, it may not be amiss to have such a table as the following; by which they may be found when logarithmic tables are not at hand. The multipliers are all calculated as above, for an hypothenusal line of 1000 links or 100 chains.

A TABLE for reducing HYPOTHENUSAL LINES to HORIZONTAL LINES.

Angles.	Multipliers.	Angles.	Multipliers.	Angles.	Multipliers.
3° 00′	,9987	6° 20′	,9939	9° 40′	,9858
3 10	,9985	6 30	,9936	9 50	,9853
3 20	,9983	6 40	,9933	10 00	,9848
3 30	,9981	6 50	,9930	10 10	,9843
3 40	,9979	7 00	,9926	10 20	,9838
3 50	,9977	7 10	,9922	10 30	,9833
4 00	,9975	7 20	,9918	10 40	,9827
4 10	,9973	7 30	,9914	10 50	,9822
4 20	,9971	7 40	,9910	11 00	,9816
4 30	,9969	7 50	,9906	11 10	,9811
4 40	,9967	8 00	,9902	11 20	,9805
4 50	,9965	8 10	,9898	11 30	,9799
5 00	,9962	8 20	,9894	11 40	,9793
5 10	,9960	8 30	,9890	11 50	,9787
5 20	,9957	8 40	,9886	12 00	,9781
5 30	,9954	8 50	,9882	12 10	,9775
5 40	,9951	9 00	,9877	12 20	,9769
5 50	,9948	9 10	,9873	12 30	,9763
6 00	,9945	9 20	,9868	12 40	,9757
6 10	,9942	9 30	,9863	12 50	,9750

A TABLE of HYPOTHENUSAL reduced to HORIZONTAL LINES.

Angles.		Multipliers.	Angles.		Multipliers.	Angles.		Multipliers.
13°	00′	,9744	16°	20′	,9597	18°	50′	,9465
13	10	,9737	16	30	,9588	18	55	,9461
13	20	,9730	16	40	,9580	19	00	,9456
13	30	,9724	16	50	,9571	19	05	,9450
13	40	,9717	17	00	,9563	19	10	,9446
13	50	,9710	17	10	,9554	19	15	,9441
14	00	,9703	17	20	,9546	19	20	,9436
14	10	,9696	17	30	,9537	19	25	,9431
14	20	,9689	17	40	,9528	19	30	,9426
14	30	,9682	17	50	,9519	19	35	,9421
14	40	,9675	18	00	,9510	19	40	,9416
14	50	,9668	18	05	,9506	19	45	,9411
15	00	,9660	18	10	,9501	19	50	,9406
15	10	,9653	18	15	,9497	19	55	,9401
15	20	,9645	18	20	,9492	20	00	,9396
15	30	,9637	18	25	,9488	20	05	,9391
15	40	,9629	18	30	,9483	20	10	,9386
15	50	,9621	18	35	,9479	20	15	,9381
16	00	,9613	18	40	,9474	20	20	,9376
16	10	,9605	18	45	,9470	20	25	,9371

A TABLE of HYPOTHENUSAL reduced to HORIZONTAL LINES.

Angles.	Multipliers.	Angles.	Multipliers.	Angles.	Multipliers.
20° 30′	,9366	22° 10′	,9260	23° 50′	,9147
20 35	,9361	22 15	,9255	23 55	,9141
20 40	,9356	22 20	,9249	24 00	,9135
20 45	,9351	22 25	,9244	24 05	,9129
20 50	,9346	22 30	,9238	24 10	,9123
20 55	,9341	22 35	,9233	24 15	,9117
21 00	,9336	22 40	,9227	24 20	,9111
21 05	,9331	22 45	,9222	24 25	,9105
21 10	,9326	22 50	,9216	24 30	,9099
21 15	,9321	22 55	,9211	24 35	,9093
21 20	,9315	23 00	,9205	24 40	,9087
21 25	,9310	23 05	,9200	24 45	,9081
21 30	,9304	23 10	,9194	24 50	,9075
21 35	,9298	23 15	,9189	24 55	,9069
21 40	,9293	23 20	,9183	25 00	,9063
21 45	,9288	23 25	,9177	25 05	,9057
21 50	,9282	23 30	,9171	25 10	,9051
21 55	,9276	23 35	,9165	25 15	,9045
22 00	,9271	23 40	,9159	25 20	,9038
22 05	,9266	23 45	,9153	25 25	,9032

A TABLE of HYPOTHENUSAL reduced to HORIZONTAL LINES.

Angles.		Multipliers.	Angles.		Multipliers.	Angles.		Multipliers.
25°	30′	,9026	27°	05′	,8903	28°	35′	,8771
25	35	,9020	27	10	,8897	28	40	,8774
25	40	,9014	27	15	,8890	28	45	,8767
25	45	,9008	27	20	,8883	28	50	,8760
25	50	,9001	27	25	,8877	28	55	,8753
25	55	,8995	27	30	,8870	29	00	,8746
26	00	,8989	27	35	,8863	29	05	,8739
26	05	,8983	27	40	,8856	29	10	,8732
26	10	,8976	27	45	,8850	29	15	,8725
26	15	,8970	27	50	,8843	29	20	,8718
26	20	,8963	27	55	,8836	29	25	,8711
26	25	,8957	28	00	,8829	29	30	,8704
26	30	,8950	28	05	,8823	29	35	,8697
26	35	,8944	28	10	,8816	29	40	,8689
26	40	,8937	28	15	,8809	29	45	,8682
26	45	,8931	28	20	,8802	29	50	,8674
26	50	,8924	28	25	,8795	29	55	,8667
26	55	,8917	28	30	,8788	30	00	,8660
27	00	,8910						

The Use of this TABLE.

If the angle of elevation is in it, multiply the up-hill line by the number anſwering the angle; if not, by that anſwering the angle neareſt it: the product is the horizontal line to the foot of the altitude, or perpendicular height.

EXAMPLE I.

Let the ſame horizontal line be required as the firſt example of the problem?

$$
\begin{array}{lrl}
\text{Tabular number} & ,9446 & \text{for } 19^\circ\ 10' \\
\text{Up-hill line} \quad - & 3480 & \\
\hline
 & 75568 & \\
 & 37784 & \\
 & 28338 & \\
\hline
\text{Horizontal line} & 3287,208 & \text{the same as before.}
\end{array}
$$

EXAMPLE II.

Let it be required to find the horizontal line of example ſecond foregoing?

Up-hill line - - - - - - - 2865
Nearest tabular number to $14^\circ\ 12'$,9696

$$\begin{array}{r} 2865 \\ ,9696 \\ \hline 17190 \\ 25785 \\ 17190 \\ 25785 \\ \hline 2777,904 \end{array}$$

Horizontal line - - - - 2777,904 nearly the same as before.

PROB. XII.

To find the perpendicular height of a tower, house &c. where you can measure to or from the foot of it in an horizontal line; by the quadrant.

RULE.

Measure from the tower, till you find the angle of elevation $45^\circ\ 00'$, and then the distance is equal to the height. Or, take the angle of elevation at any distance from it; measure that distance, and then say, as radius to the distance, so is the tangent of the angle of elevation to the altitude above the centre of the quadrant.

PROB. XIII.

To do the same by the chain.

RULE.

Take the angle of elevation by Prob. X. and proceed as above, with the distance of the pin from the tower, to find its height above the pin. Or, say, as the

distance of pin and pole to the difference of their heights, so is the distance of the tower from the pin to the height above it.

PROB. XIV.

Having the height of a tower &c. above a horizontal plain, to find its distance from any point in that plain; by the quadrant.

RULE.

As radius to the height, so is the tangent of the angle of depression to the distance.

Note, I suppose these three last Problems so plain, as to need no Examples to explain them.

PROB. XV.

To measure an inaccessible height, as a mountain &c. from a plain, where you can go back from the foot of it, or forward to it; by the quadrant.

RULE.

Take the angle of elevation any where upon the plain; measure, back or forward, a considerable distance, and take the angle again: then, as the sine of the difference of the angles taken, to the distance measured, so is the sine of the least angle to a 4th term: and as radius to this 4th term, so is the sine of the greatest angle to the altitude.

EXAMPLE.

Suppose, at the foot of a hill, I find the angle of ele-

vation 19° 30′, and measuring back 2940 links, I find it 14° 50′. I demand the altitude of the hill?

Greatest angle	- -	19° 30′
Least	- -	14 50
Difference	-	4 40

		Ar. Co.
As the sine of the difference of the angles 4° 40′ - -	Log.	1.089596
To the distance measured between, 2940	L.	3.468347
So is the sine of the least angle 14° 50′	L.	9.408254
To the 4th term, or hypothenusal line 9251 - - -	L.	3.966197

As radius sine of 90° 00′ -	Log.	10.
To the hypothenusal line 9251 -	L.	3.966197
So is the sine of the gr. angle 19° 30′	L.	9.523495
To the altitude of the hill 3088 -	L.	3.489692

Answer 3088 links.

PROB. XVI.

To measure any angle on the surface of the field; by the graphometer or quadrant

RULE.

Having placed the centre above the angular point, level so as you can see both the poles, or marks at the ends of the lines that form the angle; there screw fast:

turn the diameter, or radius, to 00° 00′ towards one end pole, and the index towards the other: then you will ſee the meaſure of the angle cut off by the central edge of the index upon the limb. But if the angle be above 90°, and the ground not level, you muſt not uſe the quadrant. When the angle is not above 90°, or the ground tolerably level, you may uſe it; for any angle, by ſetting the radius to 00° 00′, upon one end pole, that is looking over the centre, and 00° 00′ to it, and placing a pole in a right line with the index upon 90°, then bringing the radius to 00° 00′ upon this, and turning the index upon the other end pole, it will cut off the meaſure of the angle above 90°.

PROB. XVII.

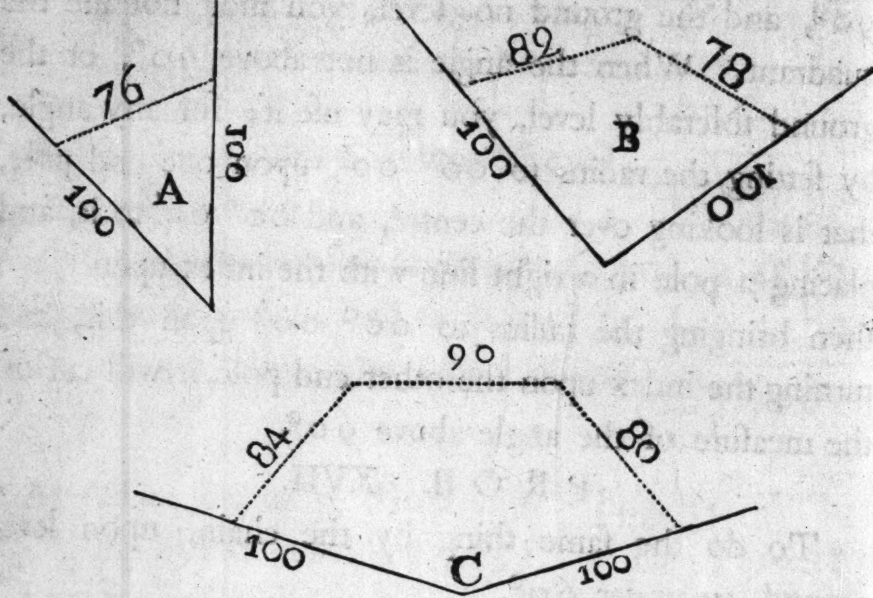
To do the ſame thing by the chain, upon level ground, or under 60°.

RULE.

Meaſure from the angular point one chain upon each of the ſides, ſetting poles or pins at the ends of theſe meaſures: if theſe poles or pins are above a chain diſtant, ſet one or two more as far from the angular point as the two firſt, and ſo as none of them all may be more diſtant than one chain: one ſet between the firſt two makes two angles, and two of them make three; find theſe angles by the following proportion, and add them together, their ſum is the angle required.

The proportion is, as 100 to radius, so is half the distance of the pins to the sine of half the angle.

EXAMPLES.



$$2)\underline{76}$$
$$38$$

For the angle A.

As 100	- -	Log.	2.
To radius	- -	L.	10.
So is 38	- -	L.	1.57978
To 22° 20′	-	L.	9.57978

$$\underline{\quad 2 \quad}$$
$$\angle A = 44\ 40$$

For the angle B.

$$2)\underline{82}$$
$$41 \quad \text{Log. } 9.61278 = \text{S. } 24^\circ\ 12' \times 2 = 48^\circ\ 24'$$
$$2)\underline{78}$$
$$39 \quad \text{L. } 9.59106 = \text{S. } 22\ \ 57 \times 2 = \underline{45\ \ 54}$$
$$\angle B = 94\ \ 28$$

For the angle C.

$$2)\underline{84}$$
$$42 \quad \text{Log. } 9.62325 = \text{S. } 24^\circ\ 50' \times 2 = 49^\circ\ 40'$$
$$2)\underline{90}$$
$$45 \quad \text{L. } 9.65321 = \text{S. } 26\ \ 45 \times 2 = 53\ \ 30$$
$$2)\underline{80}$$
$$40 \quad \text{L. } 9.60206 = \text{S. } 23\ \ 35 \times 2 = \underline{47\ \ 10}$$
$$\angle C = 150\ \ 20$$

It appears by the work for the angle A, that the logarithm of half the chord (or diſtance of the pins) with 9. for the index, is the logarithmic ſine of half the angle; and this will hold true for all the half angles above 5° 44′, and the index below that, to 34′ will be 8. Or, by adding 8 to the index of the logarithmic half chord, you will have the logarithmic ſine of half the angle. This is the reaſon of the contraction in the work for the angles B and C as above.

In taking angles on the field by the quadrant and chain, there are two conditions required, that the ground be nearly level; or that the angle do not exceed 90° for the quadrant, or 60° for the chain: the reaſon is, that the two ſides of any angle muſt be in the ſame plain; but if the ſurface of one part of an angle taken by the quadrant is higher or lower than the other part, both the ſides are not in the ſame plain; for that ſurface is the ſurface of the quadrant, and extends no farther than it does, that is only to 90° 00′, ſo that the exceſs of the meaſure is really that of another angle, not a part of the ſame. The caſe is the ſame with the chain in an angle above 60°, for you cannot be ſure of the ſame ſurface any farther than one chain's length, which is, by the rule, the radius for meaſuring the angle, and the chord of 60° is equal to the radius. But when the two ſides are nearly in the ſame plain, or in the plain of the ſurface of the quadrant, which is the level I mean; any angle whatſoever may be meaſured by it: as alſo by the chain, when all the chords, or diſtances of the poles or pins, are in the ſame plain.

There is no condition required to limit the meaſure of the angle to be taken by the Graphometer, becauſe the plain of its ſurface extends to any angle whatſoever. And if it have a radius large enough, it is the beſt inſtrument for this purpoſe, that can be uſed in the field, of all that have been as yet contrived.

PROB. XVIII.

To meaſure an inacceſſible height from a plain where you can neither go back nor forward, but only to one ſide; by the graphometer and quadrant.

RULE.

On any part of the plain, take the angle formed by a right line from your ſtation to the top of the height, and another from the ſame to a pole ſet up, where you deſign your ſecond ſtation; and leave a pole at this ſtation: meaſure to the ſecond ſtation, and there take the angle as before, at the firſt, with the pole left there; take alſo the angle of elevation. Then, as the ſine of the ſupplement of the ſum of the two firſt angles to the ſtation diſtance, ſo is the ſine of the firſt angle to the hypothenuſal line: and, as radius to the hypothenuſal line, ſo is the ſine of the angle of elevation to the height required.

EXAMPLE.

Let the firſt angle be 69° 40′, the ſtation diſtance 3460 links, the ſecond angle 53° 50′, and the angle of elevation 17° 35′. I demand the height of the hill?

$$\left.\begin{array}{rr} 69^\circ & 40' \\ 53 & 50 \end{array}\right\}\text{ two firſt angles.}$$

$$\begin{array}{rrl} \hline 123 & 30 & \text{Sum.} \\ 180 & 00 & \\ \hline 56 & 30 & \text{Supplement.} \end{array}$$

		Ar. Co.
As the ſine of 56° 30′ - -	Log.	0.078893
To the ſtation diſtance = 3460 -	L.	3.539076
So is the ſine of 69° 40′ - -	L.	9.972058
To the hypothenuſal line = 3891	L.	3.590027

As radius - - - -	Log.	10.
To the hypothenuſe = 3891	L.	3.590027
So is the ſine of 17° 35′ -	L.	9.480140
To the altitude 1175 - -	L.	3.070167

Anſwer 1175 links.

PROB. XIX.

To meaſure a right line, or diſtance, acceſſible only at one end, by the graphometer.

RULE.

Set up a pole, or obſerve a mark, at ſome diſtance from the acceſſible end, to which you can meaſure from it: take the angle formed by the inacceſſible diſtance, and that to be meaſured: meaſure it, and take the angle formed by the meaſured line, and the line from its end to the inacceſſible end. Then, as the ſine of the ſupplement of the ſum of theſe angles to the meaſured diſtance, ſo is the ſine of the ſecond angle to the inacceſſible diſtance.

EXAMPLE.

Standing by a river's ſide, I obſerve a tree on the other ſide, and cauſe a pole to be ſet up on this ſide; I find the firſt angle (taken as above) 63° 25′, the diſtance to the pole 1760 links, and the angle there 56° 15′. I demand the diſtance from my firſt ſtation to the tree beyond the river?

$$\left.\begin{array}{rr} 63^\circ & 25' \\ 56 & 15 \end{array}\right\} \text{Angles taken.}$$

119	40	Sum.
180	00	
60	20	Supplement.

		Ar. Co.
As the ſine of 60° 20′	- -	Log. 0.061019
To the ſtation diſtance = 1760	-	L. 3.245513
So is the ſine of 56° 15′	- - -	L. 9.919846
To the required diſtance = 1684	-	L. 3.226378

Anſwer 1684 links.

PROB. XX.

To perform the ſame thing by three poles, without taking angles.

RULE.

Set the firſt pole at the acceſſible end, and from it

raiſe a perpendicular to the inacceſſible line of any conſiderable length; at the end of this, raiſe a perpendicular to the line from it to the inacceſſible end, and ſet the ſecond pole; let this perpendicular juſt reach to the inacceſſible line produced from the firſt pole, and there ſet the third pole. Divide the ſquare of the firſt perpendicular by the diſtance of firſt and third pole, the quotient is the inacceſſible line.

EXAMPLE.

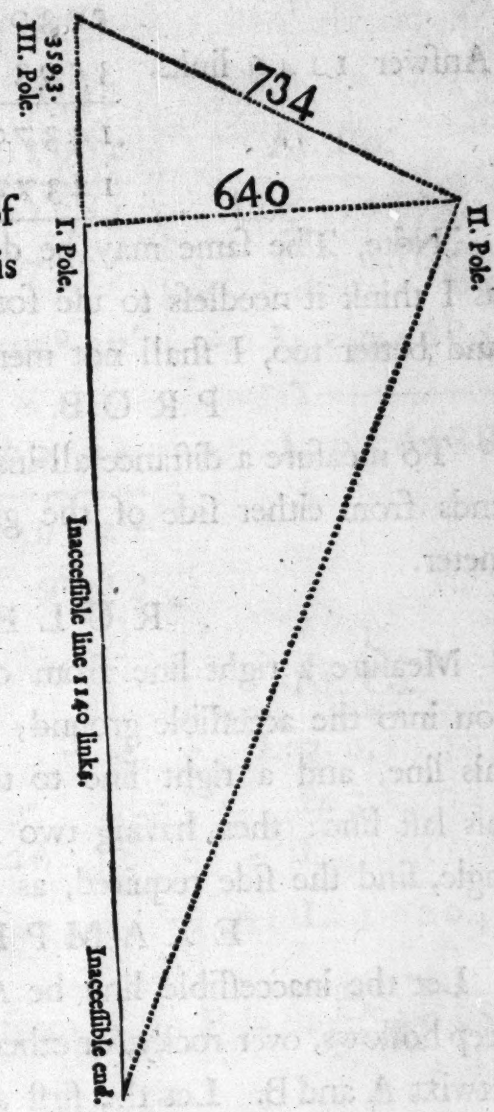
Let the firſt perpendicular be 640, the ſecond 734, and the diſtance of firſt and third pole, conſequently, 359 links. I demand the inacceſſible diſtance?

Firſt perpendicular - -	640
	640
	256
	384
Square - - - - -	409600
Second perpendicular - - -	734
	734
	2936
	2202
	5138
Square - - - - - -	538756
Square of firſt perpendicular -	409600
Difference of the ſquares -	129156

Difference of squares 129156(359,3

```
      9
   ------
65) 391
    325
   -------
709) 6656
     6381
    ------
      275
```

Thus the distance of first and third pole is found 359 links.



Square of first perpendicular

Distance of the first and third pole $= 359,3)409600,0(1140$ = Inaccessible line.

$$
\begin{array}{r}
3593 \\
\hline
5030 \\
3593 \\
\hline
14370 \\
14372 \\
\hline
\end{array}
$$

Answer 1140 links.

Note, The same may be done by four poles: but, as I think it needless to use four, when three can serve, and better too, I shall not mention that method.

PROB. XXI.

To measure a distance all inaccessible, except the two ends from either side of the ground, by the graphometer.

RULE.

Measure a right line from one end that will bring you into the accessible ground; take the angle made by this line, and a right line to the other end; measure this last line: then having two sides, and the included angle, find the side required, as below.

EXAMPLE.

Let the inaccessible line be AB, going thro' many steep hollows, over rocky, or otherwise inaccessible places, betwixt A and B. Let the first accessible line be AC=

1965 L. the angle at C = 121° 26′, and the second line CB = 9320 L. Required AB?

See Plate I. Fig. 2.

$$
\begin{array}{l}
180^\circ\ 00' \\
\underline{121\ \ \ 26} \\
2)\ \ 58\ \ \ 34\,(29^\circ\ 17'
\end{array}
$$

As CB + CA =	9320 1965 11285		Ar. Co. Log. 5.94750
To CB − CA =	7355	-	L. 3.86658
So is T. $\frac{\angle A + \angle B}{2}$ =	29° 17′	-	L. 9.74880
To T. $\frac{\angle A - \angle B}{2}$ =	20 04	-	L. 9.56288
$\angle A$ =	49 21		
$\angle B$ =	9 13		

		Ar. Co.
As S. $\angle A$ = 49° 21′	- - -	Log. 0.11993
To CB = 9320	- - - - -	L. 3.96942
So is S. $\angle C$ = 121° 26′	- - -	L. 9.93108
To AB = 10482	- - - -	L. 4.02043

Thus the distance from A to B is found 10482 links.

PROB. XXII.

To produce or continue a right line, thro' accessible ground, where you lose sight of both ends.

RULE.

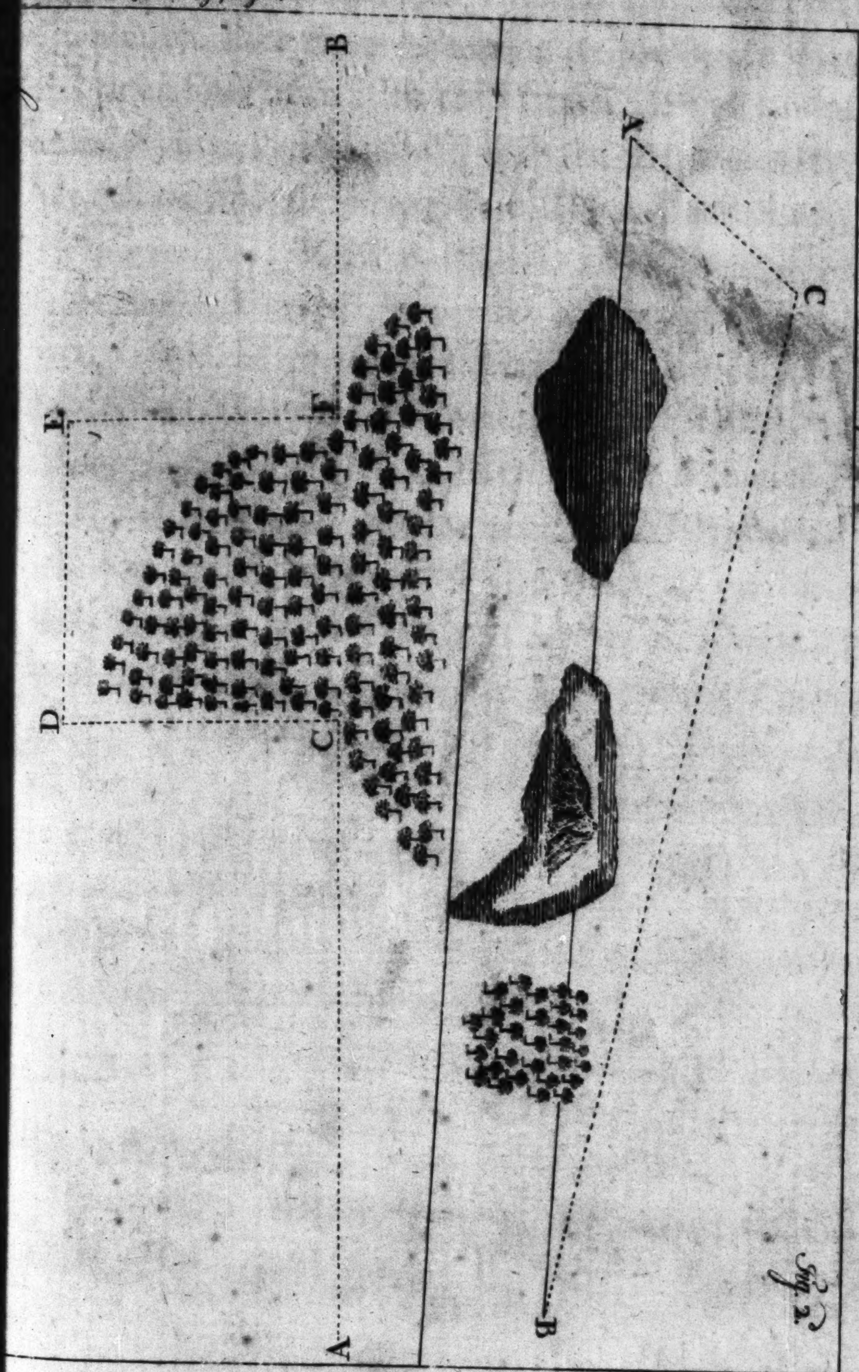
When you observe that you are like to lose sight of both ends, set a pole at any part of the chain; let the foremost chain-bearer direct himself by this, till you are like to lose sight of it too; then set another pole, and so on as before. But take care, in setting these poles, that they be exactly upon the true tract of the chain; for when they are often set, a very small deviation at any time will be a great error, if the line is long. If there is any conspicuous mark right in a line with the chain, it will do as well for the hindmost chain-bearer to direct the other by it, if it be before you, or the foremost to direct himself, if it is behind you.

Note, In taking angles when the end poles are far distant, so as to be hardly visible; poles set as above, about 6 or 8 chains from the angular point, as you are measuring the lines, will serve the purpose, as well as the end poles: or you may use the marks spoke of above, if they are small, or at a great distance.

PROB. XXIII.

To measure a distance every way inaccessible, but both ends visible; by the graphometer.

Plate I. fronting page 48.



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RULE.

Chuſe two ſtations at a conſiderable diſtance: at the firſt ſtation, take the angles made by the ſtation diſtance and right lines from it to the inacceſſible ends: meaſure to the ſecond ſtation, and there take the angles, as before: then find the inacceſſible diſtance, as below.

EXAMPLE.

Suppoſe A and B, two trees on the other ſide of a river, C and D your two ſtations on this ſide, 1344 links diſtant, the angle ACD=82° 45′, BCD=54° 20′, ADC=39° 40′, and BDC=62° 10′. Required AB?

See Plate II.

$$\begin{aligned} \angle ACD &= 82^\circ 45' \\ \angle BCD &= 54\ \ 20 \\ \hline \angle ACB &= 28\ \ 25 \end{aligned}$$

			Ar. Co.
As ſine of $\angle CAD = 57^\circ 35'$	-	Log.	0.07357
To DC = 1344	- - - - -	L.	3.12840
So is ſine of $\angle ADC = 39^\circ 40'$	-	L.	9.80504
To AC = 1016	- - - - -	L.	3.00701

$$\begin{array}{rrr} & 180^\circ & 00' \\ \angle ACB = & 28 & 25 \\ \hline 2) & 151 & 35 \\ \hline & 75 & 48 \end{array} \qquad \begin{array}{rrr} \angle ADC = & 39^\circ & 40' \\ \angle ACD = & 82 & 45 \\ \hline & 122 & 25 \\ & 180 & 00 \\ \hline \angle CAD = & 57 & 35 \end{array}$$

$$
\begin{array}{rr}
\angle BCD = & 54^\circ\ 20' \\
\angle BDC = & 62\ \ 10 \\
\hline
 & 116\ \ 30 \\
 & 180\ \ 00 \\
\hline
\angle CBD = & 63\ \ 30
\end{array}
$$

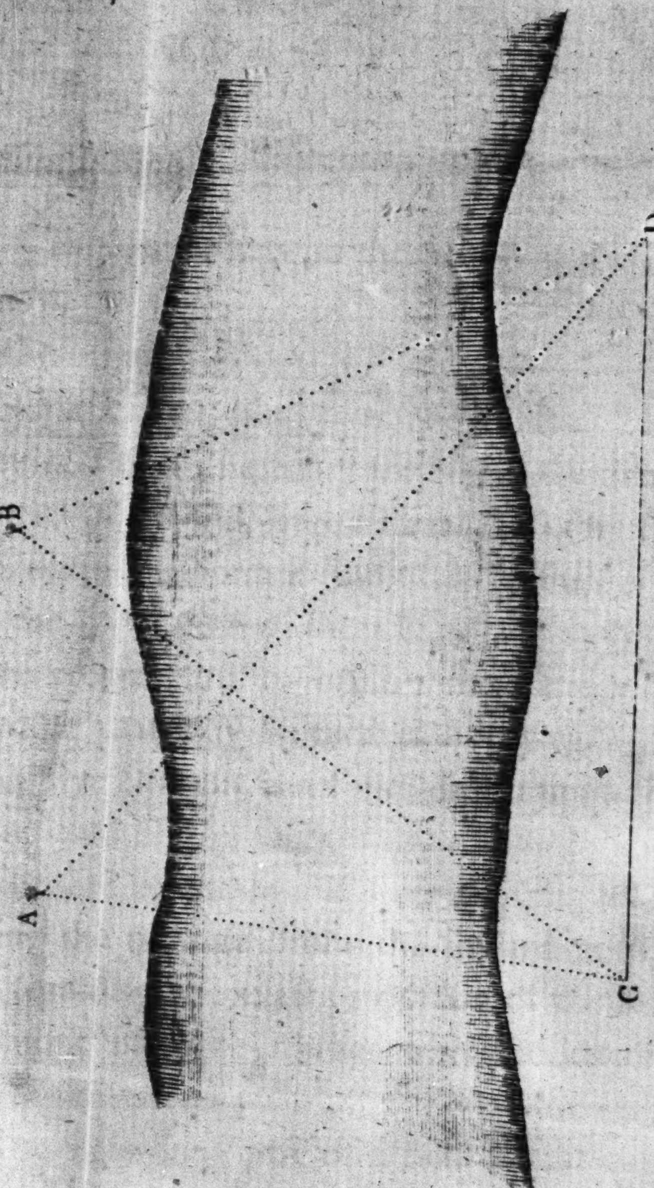
		Ar. Co.
As sine of $\angle CBD = 63^\circ 30'$ -	Log.	0.04820
To CD = 1344 - - - - - -	L.	3.12840
So is sine of $\angle BDC = 62^\circ 10'$ -	L.	9.94660
To BC = 1328 - - - - - -	L.	3.12320

$$
\begin{array}{r}
1328 \\
1016 \\
\hline
\end{array}
$$

As BC + AC = 2344 - -	Log.	3.36996
To BC − AC = 312 - - - -	L.	2.49415
So is T. $\frac{\angle CAB + CBA}{2} = 75^\circ 48'$	L.	10.59681
		13.09096
To T. $\frac{\angle CAB - CBA}{2} = 27^\circ 45'$	L.	9.72100

$\angle ABC = 48\ \ 03$

Plate II. fronting page 50.



As ſine of $< ABC = 48^\circ 03'$	-	Log.	9.87141
To $AC = 1016$	- - - -	L.	3.00701
So is ſine of $< ACB = 28^\circ 25'$	-	L.	9.67982
			12.68683
To $AB = 653{,}8$	- - - - -	L.	2.81542

Thus the diſtance betwixt the trees A and B is found 654 links.

Thus far of meaſuring lines and angles. Theſe are all the rules, I think, neceſſary for taking the dimenſions of any tract of ground, large or ſmall, in order either to find its content, or form a plan of it. I ſhall now proceed to the application of them in actual ſurveying: but it will be proper, firſt to explain ſome terms uſed by ſurveyors, or that may be uſed in ſurveying.

I. An Eye Draught is a figure drawn upon the ſpot, reſembling that of the field or ground to be meaſured in the ſituation of the ſeveral ſides and diagonals, diſtinguiſhing the outward from the inward angles, but without regarding the exact proportion of the ſides and angles to one another; marked with the letters A, B, C &c. for the ſeveral poles or marks at the corners.

II. The right-lined part of a field is that contained within the right lines from pole to pole, and repreſented by the eye draught.

III. The Off-ſets are theſe parts of a field contain-ed within the parts of the boundary betwixt the tops or ends of perpendiculars from the right line you are meaſuring, theſe perpendiculars and the parts of the right line between them. As theſe parts of the boundary muſt be ſo very near right lines, as to be ſafely taken for ſuch, all off-ſets are either triangles, or rectangles, or trapezoids; that is, quadrilateral figures having two ſides parallel, viz. the two perpendiculars from the right-lined ſide of the field.

IV. The Field Book is ſome ſheets of ſtrong paper pin'd or ſtitched up in an octavo form, ruled as you may ſee in the following examples, and for the uſes there explained.

PROB. XXIV.

To meaſure a rig or two together, called Runrig meaſure.

RULE I.

When the content only is required, which is the most common caſe. Take the breadth at one end; meaſure one chain in a perpendicular to this breadth, raiſed by the off-ſet ſtaves, or by the eye, from any convenient part of it: at the end of the chain, take the breadth again in a perpendicular to it: meaſure forward another chain &c. till you come to the other end; there take the breadth alſo; and you have done. If the furrow

are not ſtraight for the whole length of a chain, take the breadth at 50. 30 &c. ſo as to have ſtraight furrows always betwixt your breadths; mark the lengths and breadths in the field book, as you ſee below, under L. and B. when it begins or ends in a point, mark 00. under B. If the chain runs out of the rig, as it will ſometimes, when very crooked, obſerve what is directed in the next rule, when this caſe is ſuppoſed to happen.

EXAMPLE.

Two rigs laboured by J. WHITE, Anno 1756.

Left Hand.	L.	B.	Right Hand.
	00	56	
	100	64	
	200	70	
	300	68	
	40	72	
	60	78	
	500	74	
	70	80	
	30	84	
	700	84	
	800	78	
	900	78	
	50	78	
	50	70	
	1100	62	
	1200	50	
	64	00	1264 = whole length.

Note, If the breadth be more than one chain, measure, as above, first down one, then up another. You may do the same with a third and fourth rig &c.

RULE II.

If it be required to insert the rigs into a plan, set a pole at the end you measure to, and direct the chain upon it, taking the breadths all along, as above. If this line to the pole runs without the ground you are measuring, subtract the part without from the whole breadth, set the remainder under B in the field book, and the part without in a line with it, marking whether it is to the right or left hand of the ground as you are measuring. Do the same with the parts of each breadth on each side of the line, when it is all within the ground.

EXAMPLE.

One rig laboured by D. BLACK, 1756.

Left Hand.	B.	L.	B.	Right Hand.
	00	00	00	
	00	100	42	
	16	200	28	
	20	60	24	
	27	40	17	
	35	400	12	
	39	500	7	
	43	600	00	
	41	700		
6	+36	20		
19	+37	80		
30	+43	900		
20	+42	1000		
15	+39	1100		
9	+39	1200		
	45	1300		
	33	1400		
	00	85		
				1485.

Note, Tho' all the breadths are ſet down links, yet halfs and quarters, and even tenths, ſhould be taken.

PROB. XXV.

To make an eye draught of a field.

RULE.

When you are to meaſure any piece of ground, firſt walk round it, and thro' ſuch parts of it, as can give you ſome idea of its figure and ſurface, that you may know what inſtruments you will have uſe for. When you are to begin the ſurvey, being provided with every thing neceſſary, a ſufficient number of aſſiſtants, and one to ſhow the boundaries; on your field book draw a figure like that of the field, the ſides all joining the ſame way, the firſt pole, or corner where you begin, marked A, the next B &c. If you are to uſe the chain only, draw the diagonals as they are to be meaſured: if you are to take angles, diſtinguiſh carefully the outward and inward: if the field has no more than 6 ſides, this may be done, and all the poles ſet, before you begin; if no more than 10, it may, as you go round: but if more than 10 ſides, it cannot be done at all in the field book with any exactneſs: however, always mark the ſides and angles in an alphabetical order, that when the ſurvey is made, you may form a rough draught of it, which will ſerve inſtead of an eye draught, to ſhow the order in which the parts and ſeveral triangles ly, which, tho' not very exact, may direct you both in finding the ſeveral contents that make up the whole, and in forming the plan.

When there are many ſides, two poles ſtanding at one time are ſufficient, tho' more ſhould be carried a-bout, that they may be ready at hand when required.

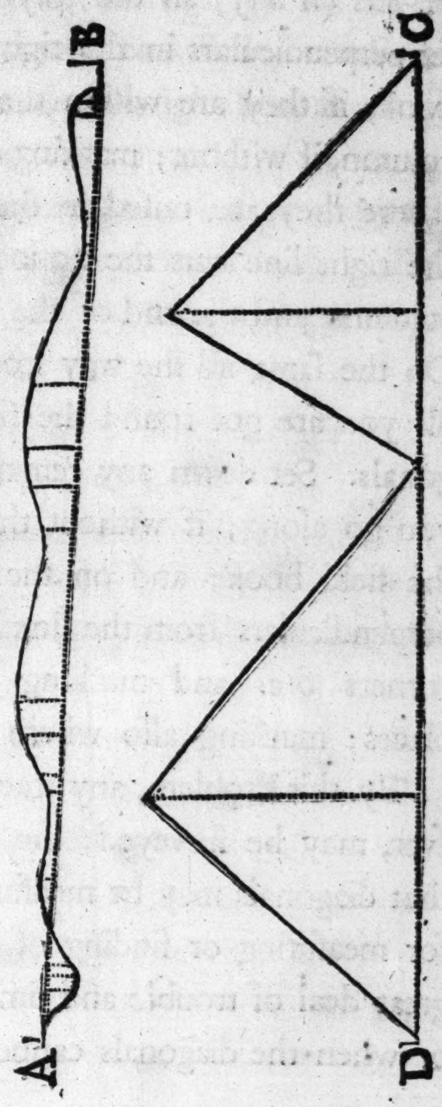
PROB. XXVI.

To meaſure the off-ſets of a field.

RULE.

As you meaſure along from one pole to another, obſerve the boundary, how far it may be reckoned a ſtraight line, and where it turns, raiſe a perpendicular from any part of the chain, where it will be a true one, to the boundary. If theſe perpendiculars are ſhort, they may be raiſed by the eye, or by the off-ſet ſtaves; but, if long, they had beſt be let fall from the turning or angular point, by Prob. VII. or VIII.

EXAMPLES.



PROB. XXVII.

To meaſure or take the dimenſions of a field, by the chain only.

RULE.

Meaſure from the firſt pole to the ſecond, taking the off-ſets (if any) all the way as you go on: ſet the off-ſet perpendiculars in the right hand column of the field book, if they are within the field, and in the left hand column, if without; marking exactly the end of the links, where they are raiſed in the middle column. Where the right line cuts the boundary, ſet o o in the off-ſet columns, and the end of the link in the middle column. Do the ſame all the way from ſecond to third pole &c. till you are got round the field; then meaſure the diagonals. Set down any remarkable thing you obſerve as you go along; if without the field, on the left ſide of the field book; and on the right, if within it; raiſing perpendiculars from the line you are meaſuring to their corners &c. and marking the lengths in the proper places; marking alſo where raiſed.

By this Problem, any tract of ground, how large ſoever, may be ſurveyed; the only condition required is, that diagonals may be meaſured by any of the Problems for meaſuring or finding of right lines, without a very great deal of trouble and time: when theſe are required, or when the diagonals cannot be meaſured at any rate;

then, and in no other caſe, angles muſt be taken, in order to calculate the other neceſſary dimenſions.

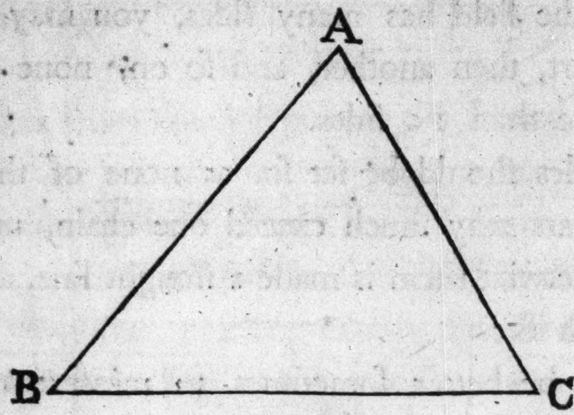
When the field has many ſides, you may meaſure firſt one part, then another, and ſo on, none of them having more than 10 ſides.

The poles ſhould be ſet ſo, as none of the off-ſet perpendiculars may much exceed one chain, unleſs the boundary betwixt them is made a ſtraight line, as a wall, hedge, ditch &c.

It may be better ſometimes to meaſure diagonals before ſome of the ſides, for quickneſs and diſtinct work.

EXAMPLE I.

New-Mains. Easter Park. Sir W. I. Anno 1756.

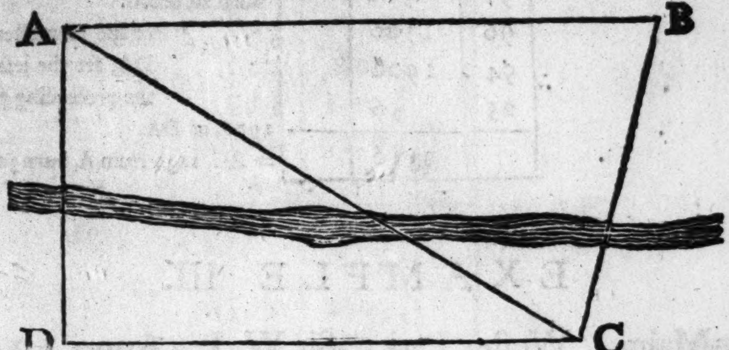


Eye Draught.

Left Hand. Remarks.	Off.	Dist.	Off.	A standing Stone. Right Hand. Remarks.
A Ditch all along from A to B.		00	32	Bushes all the way to B.
		100	48	
		200	56	
		300	64	A Tree.
		400	80	
		600	75	
		800	72	
		1000	60	
		1400	52	
		1500	40	
		1600	34	
		76	00	1676 = AB.
A Gate.	00	2060	00	2060 = BC a Park Wall.
A Hedge and Ditch.	0	00		
	42	200		
	58	400		
	66	500		
	80	600		
	73	700		
	86	800		
	74	1000		
	30	1200		
	50	1400		
	67	1800		
	40	67		1867 = CA.

EXAMPLE II.

New-Mains. Middle Park. Sir W. I. Anno 1756.

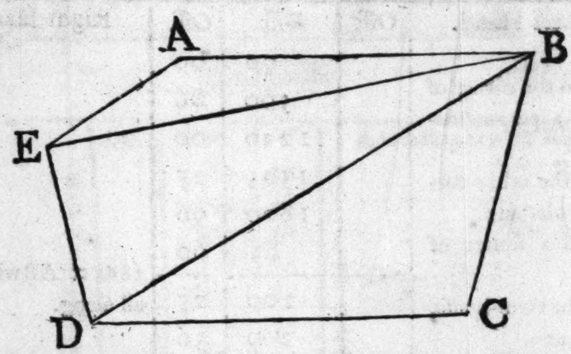


Left Hand.	Off.	Dist.	Off.	Right Hand.
		00	00	
* At 400, to the corner of the barn-yard, a perpendicular 280.		560	86	
		1220	00	
At 750, to the other corner, a perpendicular 220.		1525	25	
		1800	96	
Barn-yard is a square of 360.		63	00	1863 = AB with a row of trees all along.
At 800, to the farm-house, a perpendicular 200.		100	25	
		200	36	
At 1000, to the entry of the close, a perpendicular 174.		300	42	
		400	79	
At 1280, to the corner of ditto, a perpendicular 160.		500	60	
Close a rectangle of 520 by 360.		600	83	
		700	26	
See Plate V. in Part V.		800	52	Burn, 20 broad. Bushes on both sides of it all the way.
		900	56	
		1000	93	
		1100	52	
		1200	13	
		1300	58	
		1400	60	
		76	20	1476 = BC.
	00	2060	00	2060 = CD = BC before. Planting begins at 1660 from C, a triangle.

Left Hand.	Off.	Dist.	Off.	Right Hand.
	27	300		Plant. ends.
	97	400		Burn 24 broad.
	96	1700		* For the Homestead upon DA, see the left side of the preceeding page.
	94	1900		
	25	56		1956 = DA.
		2540		= AC. 1237 from A, burn 32 broad.

EXAMPLE III.

New-Mains. Wester Park. Sir W. I. Anno 1756.

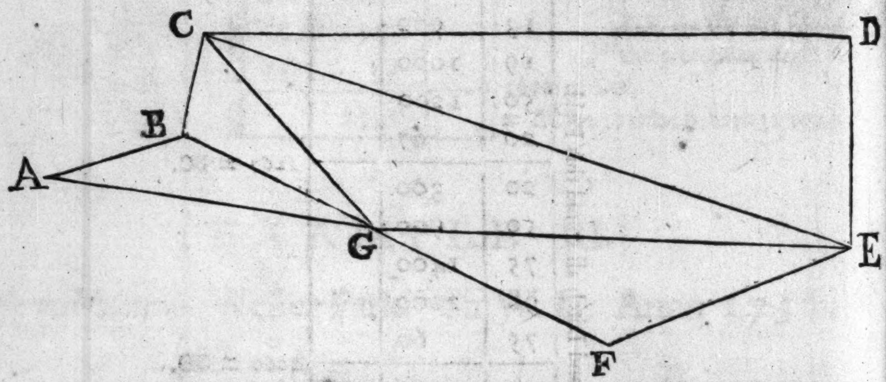


Left Hand.	Off.	Dist.	Off.	Right Hand.
A Hedge and Ditch all round.		00	00	
		100	27	
		300	79	At 800 from A, a straight row of trees to 300 from D.
		900	75	
		1200	73	
		1600	26	
		73	24	1673 = AB.
		100	27	
		200	36	
		300	75	
		400	20	
	00	500	00	

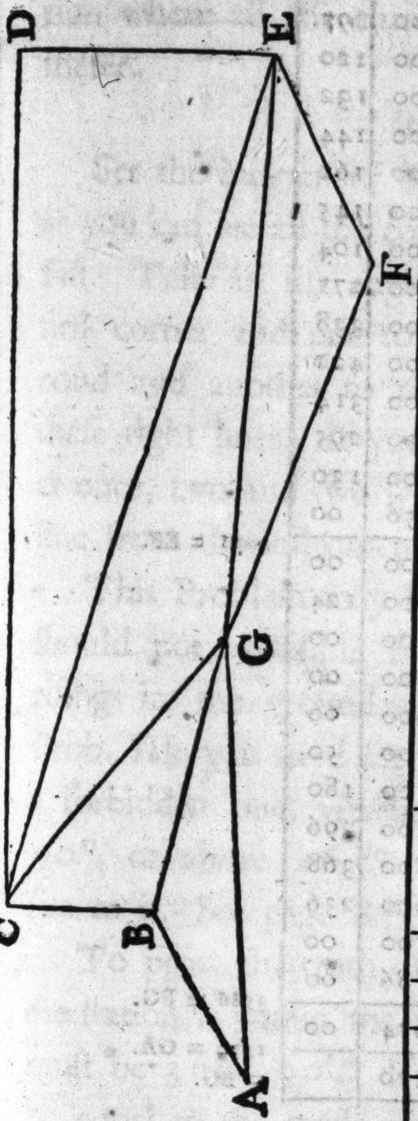
Left Hand.	Off.	Dist.	Off.	Right Hand.
Hedge and Ditch all round this Field.	27	600	00	
	13	700		
	29	800		
	13	900		
	19	1000		
	50	1200		
	20	67		1267 = BC.
	20	300		
	50	900		
	75	1400		
	98	2000		
	75	60		2060 = CD.
		200	75	
		500	96	
		800	67	
		1100	56	
		1400	90	
		1700	63	
		80	20	1780 = DE.
	00	1863	00	1863 = EA = AB before.
		2800		= EB.
		2700		= BD.

EXAMPLE IV.

The Black Common. Anno 1756.



Left Hand.	Off.	Dist.	Off.	Right Hand.
		00	00	
		100	73	
		300	60	
		500	97	
		700	83	
		900	150	
		1000	198	
		1200	168	
		1400	94	
		1800	54	
	00	50	00	
	28	50		
	73	2000		
	84	2200		
	00	80		2280 = AB.
	00	100	00	
	78	200		
	150	300		
	265	500		
	127	700		



Left Hand.	Off.	Dist.	Off.	Right Hand.
	00	30	00	
		70	26	
		900	24	
		1500	00	
		48	00	1548 = BC.
	00	100	00	
	82	200		
	146	500		
	250	600		
	280	900		
	157	1200		At 1598, to the side of a round lake, a perpendicular is 500.
	120	1700		
	00	20	42	
		80	70	
		2000	158	
		2400	173	
		2700	150	
		3000	248	
		3600	357	
		4000	480	
		4700	525	
		4900	260	
		38	00	4938 = CD.
	00	00	00	
		732	296	
		2184	00	2184 = DE.
		5720		= EC.
		2867		= CG.
		6176		= GE a high road for 400, which cuts it again at 2780. from G, from within.
	00	00	48	
		100	56	
		200	64	
		300	73	

E

Left Hand.	Off.	Dist.	Off.	Right Hand.
		90	80	
		10	89	
		500	97	
		600	120	
		700	132	
		800	144	
		900	162	
		1000	145	
		1500	104	
		2000	271	
		2600	338	
		3000	420	
		3300	314	
		3900	295	
		4000	130	
		56	00	
	00	700	00	4056 = EF.
	00	900	124	
	36	1000	00	
	42	1200	00	
	00	1600	00	
		1700	50	
	00	2100	180	
		2700	396	
		2800	368	
		3000	236	
		3500	00	
		86	00	
	00	1974	00	3586 = FG.
		1570		1974 = GA. = BG.

PROB. XXVIII.

To ſurvey a field having no off-ſets, from one ſtation where all the corners are viſible, by the graphometer.

RULE.

Set the inſtrument upon the ſtation, and level it ſo, as you can ſee all the corners thro' the ſights, then ſcrew faſt. Take all the angles, made by a right line to the firſt corner and one to the ſecond, by one to the ſecond and another to the third &c. then meaſure all theſe right lines. If you cannot level to all the corners at once, two and two will ſerve. You may meaſure one line from the inſtrument, and another to it &c.

This Problem may be applied, tho' all the corners ſhould not be viſible from your ſtation, by reaſon of riſings in the ground: for, in that caſe, poles ſet by Prob. III. will ſerve for them. But when any angle is a forbidden one, viz. under 15°, or within 15° of 90°, or above 165°, meaſure the ſide that is oppoſite to it. See page 8th.

To prove the truth of the angles, obſerve, that when the ſtation is within the field, the ſum of all the angles muſt be 360° 00′. If in one corner, that ſum muſt be equal to the angle contained betwixt the two adjacent ſides, or to its ſupplement to 360° 00′, if it be an outward angle.

EXAMPLE I.

Deer Park. Station I. within.

AIB	60° 50′	640	=IA
BIC	56 16	950	=BI
CID	52 45	600	=IC
DIE	47 26	520	=DI
EIF	45 25	560	=IE
FIG	49 48	610	=FI
GIH	16 50	590	=IG
HIA	30 40	480	=HI

EXAMPLE II.

Crow Park. Station A. One Angle. In.

BAÇ	38° 40′	320	= AB
CAD	54 28	630	= CA
DAE	51 45	710	= AD
EAF	24 30	680	= EA
		340	= AF

Note, The work of this and the following Problem may be done by the chain, taking the angles by Problem XVII.

PROB. XXIX.

To ſurvey any field, going round it, by the graphometer.

RULE.

Place the inſtrument at one angle, and meaſure from another to it, taking the off-ſets, and noting down the remarks all the way as you go on: when you come to the inſtrument, take the angle formed by the firſt meaſured ſide and the next ſide, meaſure that next ſide &c. till you are got quite round; diſtinguiſhing carefully the inward and outward angles. But meaſure the right lines ſubtending the forbidden angles.

EXAMPLE.

Sheep Park. J. H. Eſq. Anno 1756.

		00	52	
		100	64	
		200	48	
		300	30	
		400	22	
One row of trees all around from A by H, G, F &c. to B.		500	6	
$BAb = 30^\circ\ 30'$. $BAc = 57^\circ\ 40'$		600	00	
$ABb = 16\ \ 20$. $ABc = 48\ \ 00$		864	00	
Hillocks from *b*, by *c* and D to E.				864 = AB.
fgno a pond. A ſtream from it to *m*.	ABC	$73^\circ\ 30'$	In.	
bil a grove.	00	696	00	696 = BC.

mEo = 56° 00′
Emo = 92 05
mEn = 51 10
Emn = 98 45
} *Em* = 480

kFl = 91 50
Fkl = 35 20
kFi = 45 00
Fki = 97 00
} *Fk* = 474

He = 826. *ied* = 127° 15′
edi = 28 20
deh = 47 00
edh = 69 00
} *de* = 368

Adg = 67° 30′. *Adf* = 60° 00′
dAg = 51 10. *dAf* = 58 40

See Part V. Prob. XV. and Plate VIII.

BCD	149° 27′	In.	
00	00	00	
168	560		
00	920		920 = CD.
CDE	152° 20′	In.	
	00	00	
	432	260	
	610	00	610 = DE.
DEF	123° 28′	In.	
00	1027	00	1027 = EF.
EFG	164° 27′	In.	
	00	84	
	920	00	920 = FG.
FGH	120° 50′	In.	
00	570	00	570 = GH.
GHA	70° 38′	In.	
00	1802	00	1802 = HA.
HAB	134° 40′	Out.	

This field having eight ſides, the ſum of all the inward angles, with the ſupplement of the outward angle to 360° 00′, muſt be equal to 12 right angles, by Euclid I. 32. Theor.

An eye draught not being of any ſervice, it is not uſed in the laſt three examples; but the uſe of it is referred to the rough draught, which ought always to be made when you uſe theſe Problems.

The foregoing Problems, I think, are ſufficient for meaſuring any tract of land, or any field, how large or how ſmall ſoever, and of whatever ſhape or figure it be (excepting the circle and oval) in order both to find the content, and form the plan of it. Such as are uſed for planning only are reſerved for the Fifth Part, as properly belonging to it.

The diameters of the circle and oval are what only need to be meaſured for finding their contents. Theſe are figures that don't come every day in a Surveyor's way.

I ſhall now conclude this Firſt Part, by ſhowing how you may find the diameter of the earth, which, ſuppoſing it to be a true globe, is all that is neceſſary to find the content of its whole ſurface.

This may be done ſeveral ways. You may take the following two rules; and in the uſe of both, obſerve, that the angles ſhould be taken to a quarter of a minute, or more exactly, if poſſible.

The ſun's meridian altitude is the angle of his greateſt elevation any day at noon, or the height of his centre above the horizon, when he comes to the meridian of any place.

PROB. XXX.

To find the diameter of the earth.

RULE. I.

Find the altitude of ſome very high hill near the ſea (by Prob. XV. or XVIII.) above the level of the ſhore, with the greateſt exactneſs. Go up to the top of the hill, and there, as exactly as poſſible, take the angle formed by the altitude and a tangent to the ſurface of the ſea, or the angle of depreſſion of the ſea below the hill. Then, as radius to the altitude, ſo is the tangent of the angle of depreſſion to a 4th term, which you may call the baſe: add the ſquares of the baſe and altitude, and extract the ſquare root of the ſum; to this root add the baſe, the ſum is the tangent to the ſea. Divide the ſquare of the tangent by the altitude, and from the quotient ſubtract the altitude, the remainder is the earth's diameter.

EXAMPLE.

Let the altitude of the hill be 1540 yards, or 7 furlongs, and the angle of depreſſion 88° 48′.

As radius	- - - - -	Log.	10.
To the altitude = 7	- - -	L.	0.845098
So is T < of dep. = 88° 48′		L.	11.678878
To the baſe = 334,2	-	L.	2.523976

```
Base = 334,2
       334,2
       -----
       6684
      13368
     10026
    10026
   ---------
   111689,64 = Square of the base.

                     111689,64
Square of the alt.          49
                     ---------
                     111738,64(334,2=Sq. Root.
                      .  .  .
                      9
                     ---
                63)  217
                     189
                     ----
               664)  2838
                     2656
                     -----
               668)  18264

Square root        =  334,2
The Base           =  334,2
                      -----
Tang. to the sea   =  668,4
```

```
Tangent = 668,4
           668,4
          ------
          26736
         53472
        40104
       40104
      ----------
     7)446758,56
      ----------
       63822,65
           7,00 = Altitude.
       --------
Diameter = 63815,65 in Furlongs.

         8)63815,65
         ----------
Earth's diameter = 7976,95 in English miles.
```

Thus the diameter is found 7977 miles.

RULE II.

Four or five days before the Summer (or Winter) ſolſtice, take the ſun's meridian altitude, and obſerve ſome conſpicuous mark at a diſtance in a right line with the ſhadow of a long pole ſet perpendicular in the place where you obſerve: meaſure a horizontal line to that mark, and produce it, proving it every day at noon, till you come to the ſame time after the ſolſtice: then take the meridian altitude again. Multiply the miles meaſured

y 21600, and divide the product by the minutes in he difference of the two meridian altitudes; the quoent is the number of miles in the circumference of he earth, by which its diameter may be found, as hall be shown in the Second Part. To which I proeed.

PART II.

CALCULATION.

WHEN a field is ſurvey'd, the land-meaſure next work is, to find its content in the meaſu uſed in the country where it lies, which with us a Acres, Roods, Poles or Falls &c. This content is al called the Area, or Superficial Content.

Every area is a certain number of ſquares of t long meaſures uſed in taking the dimenſions, as ſqua links, ſquare yards &c. as in the following tables.

TABLE I.

ENGLISH LONG MEASURE.

Inches.	Links.	Yards.	Poles.	Furlongs.	Mile.
7,92	1				
36	4,54	1			
198	25	5,5	1		
7920	1000	200	40	1	
63360	8000	1760	320	8	1

TABLE II.

SCOTS LONG MEASURE.

Mile.	Furlongs.	Falls.	Ells.	Links.	Inches.
				1	8,88
			1	4,166	37
		1	6	25	222
	1	40	240	1000	8880
1	8	320	1920	8000	71040

TABLE III.

ENGLISH SQUARE MEASURE.

Links.	Yards.	Poles.	Roods.	Acres.	Mile.
20,66	1				
625	30,25	1			
25000	1210	40	1		
100000	4840	160	4	1	
64000000	3097600	102400	2560	640	1

TABLE IV.

SCOTS SQUARE MEASURE.

Mile.	Acres.	Roods.	Falls.	Ells.	Links.
				1	17,36
			1	36	625
		1	40	1440	25000
	1	4	160	5760	100000
1	640	2560	102400	3686400	64000000

PROB. I.

To reduce ſquare links to acres, roods, poles &c.

RULE.

Point off five decimal places, the integer will be t number of acres: multiply the decimals by 4, and poi off five places from the product, the integer will be t number of roods; multiply the decimals by 40, a point off the ſame number of decimals from the pr duct, the integer expreſſes the poles or falls, and the d cimals may be placed after them for the anſwer, it b ing ſeldom neceſſary to come to yards or ells: but the value of the ground, or any other reaſon requires

multiply the decimals after the poles by 30,25, and point off as before for yards, or by 30, and take in the $\frac{1}{4}$; multiply the decimals of a fall by 36, and point off &c. for ells. If there are not five places, make up that number by cyphers.

EXAMPLE I.

How many acres &c. in 3672480 ſquare links?

```
A. 36.72480
          4
   --------
R.  2.8992
         40
   --------
P. 35.968               A.   R.   P.
                Anſwer 36 :  2 :  36
```

EXAMPLE II.

How many acres, roods, falls and ells in 795842 ſquare links?

```
A. 7.95842
         4
   --------
R. 3.83368
        40
   --------
F. 33.3472
        36
   --------
    20832
   10416
   --------
E. 12.4992              A.  R.  F.   E.
                Anſwer 7 : 3 : 33 : 12½
```

EXAMPLE III.

How many acres, roods, poles and yards in 1998730 square links?

A. 19.98730
4

R. 3.94920
40

P. 37.968
$30\frac{1}{4}$

29040
242

Y. 29.282

A.	R.	P.	Y.
Answer 19 :	3 :	37 :	29.282

PROB. II.

To reduce square yards or ells to acres &c.

RULE.

Divide them by their number in an acre, for the acres; divide the remainder by their number in a rood, for roods; and the remainder by their number in a pole or fall, for poles or falls: the last remainder is the number of yards or ells over.

EXAMPLE.

How many acres &c. in 592640 square yards?

A. R. P.

484|0)59264|0(122 : 1 : 31
484
1086
968
1184
968

R.

121|0)216|0(1
121
950

P.

30,25)950,00(31
9075
4250
3025
12,25 Yards.

A. R. P. Y.

Answer 122 : 1 : 31 : $12\frac{1}{4}$

Here let it be observed, that the lines and angles, supposed to be known in the following Problems, are those that are to be measured or calculated, as necessary Dimensions.

PROB. III.

To find the area of a square.

RULE.

Multiply the side by itself.

PROB. IV.

To find the area of a rectangle or long ſquare.

RULE.

Multiply length and breadth.

PROB. V.

Having baſe and altitude, to find the content of a triangle.

RULE.

Multiply the one by half of the other.

PROB. VI.

Having baſe and altitude of a rhombus, or rhomboides, to find the area.

RULE.

Multiply them together.

I ſuppoſe here, examples and figures needleſs, to explain theſe four Problems. Remember, however to reduce the areas by Prob. I. or II. as you ſee in the following Problems.

PROB. VII.

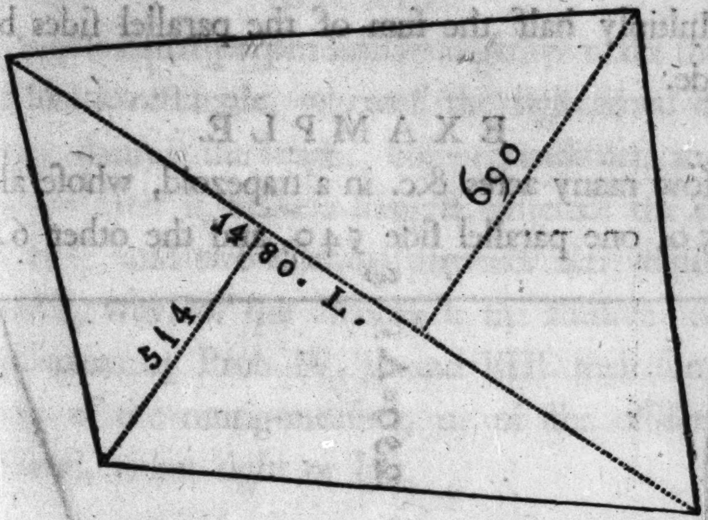
Having the diagonal and altitudes, to find the content of a trapezium.

RULE.

Multiply the diagonal by half the ſum of the altitudes.

EXAMPLE.

How many acres &c. in a quadrangular inclosure, whose diagonal is 1480 L. one altitude 690 L. and the other 514?



Altitudes = { 514	Diagonal = 1480
690	602
2)1204	296
602	888
	A. 8.90960
	4
	R. 3.6384
	40
	P. 25.536

A. R. P.

Answer 8 : 3 : 25.536

PROB. VIII.

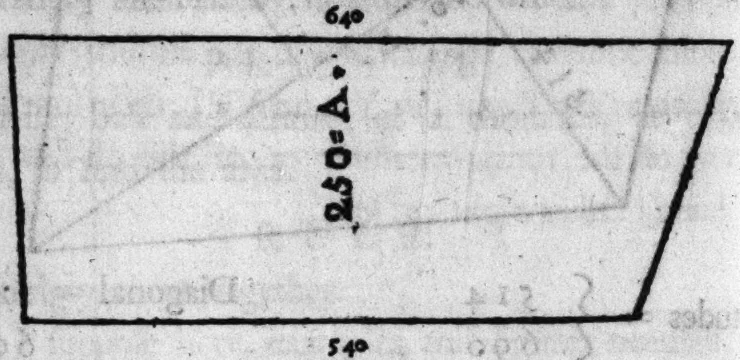
Having the two parallel ſides, and the altitude, to find the content of a trapezoid.

RULE.

Multiply half the ſum of the parallel ſides by the altitude.

EXAMPLE.

How many acres &c. in a trapezoid, whoſe altitude is 250, one parallel ſide 540, and the other 640?



Parallel ſides =	640			590
	540		Alt. =	250
	2)1180			295
	590			118
				1.47500

A. 1.47500
4
R. 1.900
40
P. 36.

A. R. P
Answer 1 : 1 : 3

PROB. IX.

To find the contents of runrig-measures, or of the off-sets of a field.

RULE.

As two equal perpendiculars together make the space or off-set a rectangle, whereof the right lined distance between them is the length; one perpendicular and ,oo, before or after it, make a triangle, whereof the distance is the base; and two unequal perpendiculars together, a trapezoid, whereof the distance is the altitude: find the several areas by Prob. IV. V. and VIII. their sum is the content of the runrig-measure, or of the off-sets upon one hand, either right or left.

EXAMPLE I.

What is the content of J. WHITE's two rigs, p. 53.

56
64
2)120
60
100
6000

64
70
2)134
67
100
6700

70
68
2)138
69
100
6900

68
72
2)140
70
40
2800

72
78
2)150
75
60
4500 &c.

6000
6700
6900
2800
4500
7600
5390
2430
8400 □
8100
7800 □
3900 □
3700
6600
5600
1600 Δ

A. 0.88020
4
R. 3.52
40
P. 20.8

R. P

Answer 3 : 2

The contents or areas marked thus □ are rectangles. That marked Δ is a triangle: the rest are trapezoids, all found much the same way as the first five. The work of which is set down. The work for the rest is omitted, because you may observe it is so easy, that the several products may be set down off-hand, without any hazard of going wrong. There are in these two rigs one triangle, three rectangles, and twelve trapezoids, as is represented by the rough draught of the annexed figure of them, and appears by the field book in p. 53. The content of the whole is, as you see, three roods, twenty poles, and $\frac{8}{10}$ of a pole.

EXAMPLE II.

How much ground is contained in the off-sets upon the side AB of Middle Park, p. 61?

```
2)86
  43

1220
  43
 366
488
52460
```

```
 1525
 1220
  305
   25
 1525
 610
2)7625
  3812
```

```
   25
   96
2)121
   60

 1800
 1525
  275
```

```
  275
   60
16500

2)96
  48
```

```
  48
  63
 144
288
3024
```

```
     52460
      3812
     16500
      3024
A. 0.75796
         4
R. 3.03184
        40
P.  1.2736
```

See Plate V. in Part V. and compare it with the field book.

R. P.
Answer 3 : 1.2736

PROB. X.

Having all the ſides of a triangle, to find the area.

RULE.

From half the ſum of the ſides, ſubtract each particular ſide; multiply the half ſum by the firſt difference, the product by the ſecond, and that product again by the third difference; extract the ſquare root of the laſt product; it is the area.

EXAMPLE I.

What is the content of the right-lined part of the Eaſter Park, p. 60?

AB = 1676	2801	1125
AC = 1866	1125	2801
BC = 2060	935	
	741	1125
2)5602		9000
		2250
2801		
		3151125
		935
		15755625
		9453375
		28360125
		2946301875

```
       2946301875
              741
     ------------
       2946301875
    11785207500
   2062411313125
   -------------- A.
   2183209689375(14.77569
   . .  .  .  .  .  .  .
24)118
    96
287)2232
    2009
2947)22309
     20629
29545)168068
      147725
       20343

295506)2034393
       1773036
2955129)26135775
        26596161

A.
Anſwer 14.77569
```

Note, You may obſerve in this and ſome of the following Examples, that when the ſum of the ſides would be an odd number, to ſhorten the work by making it even, 1, is taken from one ſide: this makes the content a very little leſs, but perhaps more true than ſtrictly obſerving the rule would do: and when one ſide is very

much longer than the others, ſhortening it, increaſeth the content; but ſtill it is truer, as the unavoidable errors in meaſuring always make the lines longer than they ſhould be: but do not ſo in ſmall triangles.

EXAMPLE II.

What is the content of the triangle CGE in the Black Common, p. 64?

```
CG=2867      7381         7381
CE=5720      4514         4514
EG=6175      1661        -----
 -----       1206        29524
2)14762                  7381
 -----                  36905
  7381                  29524
                       --------
                       33317834
```

```
  33317834          55340922274
      1661                 1206
 ----------      --------------
  33317834         332045533644
 199907004        110681844548
199907004        55340922274
33317834         --------------
-----------      66741152262444
55340922274
```

A.

```
66741152262444(81.69523
  161)274
      161
 1626)11311
       9756
16329)155552
      146961
 163385)859126
        816925
 1633902)4220124
         3267804
          952320
```

A.
Answer 81.69523.

PROB. XI.

Having all the sides and diagonals of a trapezium or irregular polygon, to find the area.

RULE.

Find the areas of the several triangles into which the figure is resolved by the diagonals, by last Problem, and add them all together, the sum is the area of the whole.

EXAMPLE I.

What is the content of the right-lined part of Middle Park, p. 61?

BC = 1476
AB = 1863
AC = 2539

2)5878

2939

2939
1463
1076
400

2939
1463

8817
17634
11756
2939

4299757
1076

25798542
30098299
4299757

4626538532
400

1850615412800

A.

18506154128oo(13.60373

23)85
69

266)1606
1596

27203)101541
81609

272067)1993228
1904469

88759

A.

13.60373 = ABC.
19.73600 = ACD.

33.33973 = ABCD.

AD = 1956	3278	4333516
CD = 2060	1322	1218
CA = 2540	1218	
2)6556	738	34668128
3278		4333516
1322		8667032
6556		4333516
6556		5278222488
9834		738
3278		4222577904
4333516		15834667464
		36947557416

A.

38953281961 44(19.73600

29)289
261

387) 2853
2709

3943) 14428
11829

2599

A.

Answer 33.33970.

EXAMPLE II.

What is the content of the right-lined part of the Black Common, p. 64, 65, 66?

```
BG = 1570
AG = 1974
AB = 2280
   ------
  2)5824
   ------
    2912
    1342
     938
     632
```

```
           2912
           1342
        -------
           5824
          11648
          8736
         2912
        -------
        3907904
            938
       --------
       31263232
      11723712
     35171136
     ----------
     3665613952
            632
     ----------
     7331227904
   10996841856
  21993683712
  -------------
  2316668017664
```

A.

23166680176664(15.22060 = ABG.

25)131
125
302) 666
604
3042) 6268
6084
30440) 1840176

BC = 1548
BG = 1570
CG = 2866
2)5984
2992
1444
1422
126

2992
1444
11968
11968
11968
2992
4320448
1422
8640896
8640896
17281792
4320448
6143677056
G

```
        6143677056
               126
       ------------
       36862062336
      12287354112
      6143677056     A.
      --------------
      7741033090 56(8.79830=BCG.
      64
      ----
  167)1341
      1169
      --------
 1749) 17203
        15741
        ---------
17588) 146230
       140704
       -----------
         552690
```

```
 DE = 2184              4237
 CD = 4938              6421
 CE = 5720              ------
      -----             4237
    2)12842            8474
      -----           16948
       6421          25422
       4237          ----------
       1483          27205777
        701
```

```
            27205777
                1483
            --------
            81617331
           217646216
          108823108
          27205777
          -----------
          40346167291
                  701
          -----------
          40346167291
      282423171037        A.
      ---------------
      282826632709991(53.18145 = CDE.
      25
      ---
103)  328
      309
      ----
1061) 1926
      1061
      -----
10628) 86563
       85024
       ------
106361) 153927
        106361
        -------
1063624) 4756609
         4254496
         --------
         .50211391
```

FG = 3586
EF = 4056
EG = 6176

2)13818

6909
3323
2853
733

6909
3323

20727
13818
20727
20727

22958607
2853

68875821
114793035
183668856
45917214

65500905771
733

196502717313
196502717313
458506340397

48012163930143

A.

```
48012163930143(69.29081 = EFG.
36
129)1201
    1161
  1382)4021
       2764
  13849)125763
        124641
  1385808)11229301
          11086464
          14283743
```

A.

```
   69.29081 = EFG.
   53.18145 = CDE.
    8.79830 = BCG.
   15.22060 = ABG.
   81.69523 = CGE. p. 92.
A. 228.18639 = ABCDEFG.
```

You might find the angles and altitude by Trigonometry, and then the area of the triangle by the Fifth Problem; but as the trigonometrical calculations will come in more properly with the Problems preparatory to the finding the contents of surveys by the graphome-

ter, and in finding these contents, I think examples of that kind needless in this place. As for measuring base and altitude upon a draught to find the content, let such as prefer guessing at the truth the easiest way to an exactness easily attained, chuse that method: it may, sometimes, come within 1 of 100; and the above methods, always, within 1 of 10000; the odds only 100 to 1, and commonly a little more, when protracted.

PROB. XII.

To find the content of any survey by the chain.

RULE.

Find the area of the right-lined part by Prob. X. or XI. (or any other, if the figure require it) find the contents of the off-sets by Prob. IX. and the difference betwixt the sum of those to the right hand and of those to the left; then if the right hand sum be greatest, add the difference to the right-lined part, the sum is the whole content: if the left hand sum be greatest, subtract, the difference &c.

EXAMPLE I.

How many acres &c. in Easter Park, p. 60?

	Left Hand Off-sets.	Right Hand Off-sets.
	4200	4000
	10000	5200
	6200	6000
107	7300	7200
67	7650	15500
749	7950	14700
642	16000	13200
2)7169	10400	22400
	8000	4600
3584	23400	3700
	3584	1290
	104684	97790

Without the field	104684	Sums of the off-sets.
Within the field	97790	
Difference - -	6894	

Right-lined part - -		14.77569 p. 90.
Difference of off-set sums		6894
	A.	14.70675
		4
	R.	2.82700
		40
	P.	33.08

A. R. P.
Answer 14 : 2 : 33

EXAMPLE II.

How many acres &c. in the Middle Park, p. 61.

Right Hand Off-sets.			Left Hand Off-sets.	
75796 on AB, p. 88.			4050	
1250	Upon the side BC.		6200	
3050			125450	
3900			19000	
6050			3330	
6950		Without	158030	Upon DA.
7150		Within	149636	
5450		Difference	8394	
3900				
5400		Right-lined part	33.33970	p. 93.
7450		Diff. of off-sets	8394	
7250				
3250		A.	33.25576	
3550			4	
5900				
3340		R.	1.02304	
149636			40	
		P.	00.9216	

A. R. P.
Answer 33 : 1 : 00.9216.

EXAMPLE III.

What is the content of the Wester Park, p. 62.

AB = 1673
AE = 1863
EB = 2800
2)6336
3168
1495
1305
368

3168
1495
15840
28512
12672
3168
4736160
1305
2368080
1420848
473616
6180688800
368
49445510400
37084132800
18542066400
2274493478400

A.

```
 .  .  .  .  .  .  .
2274493478400(15.08142 = ABE.
25)127
   125
3008)24493
     24064
 30161)42947
       30161
301624)1278684
       1206496
         72188

ED = 1780
BD = 2700
EB = 2800
   2)7280
     3640
     1860
      940
      840
```

```
        3640
        1860
     -------
     2184
    2912
    364
    ---------
    6770400
         940
   ---------
   270816
  609336
  ------------
  6364176000
         840
  ------------
  25456704
 50913408
 ---------------
 5345907840000
```

A.

```
 .  .  .  .  .  .  .
 5345907840000(23.12120 = EBD.
 4
43)134
   129
461)559
    461
4622)9807
     9244
46241)56384
      46241
      1014300



            BC = 1267
            CD = 2060
            BD = 2699
                 ----
               2)6026
                 ----
                 3013
                 1746
                  953
                  314
```

1746
3013

5238
1746
5238

5260698
953

15782094
26303490
47346282

5013445194
314

20053780776
5013445194
15040335582 A.

1574221790916(12.54700 = BCD.

22)57
44

245)1342
1225

2504)11721
10016

170579

Off-sets to the Right Hand, or within the Park.

1350
10600
46200
22200
19800
1825

101975 on AB.

1350
3150
5550
4750
1000

15800 on BC.

7500
25650
24450
18450
21900
22950
3320

124220 on DE.

Off-sets to the Left Hand, or without.

1350
2000
2100
2100
1600
6900
2345

18395 on BC.

3000
21000
31250
51900
5190

112340 on CD.
18395 on BC.

130735 = Sum.

101975 on AB.
15800 on BC.
124220 on DE.

241995 = Sum.
130735

111260 = Difference.

ABE = 15.08142
EBD = 23.12120
BCD = 12.54700

ABCDE = 50.74962 = Right-lined Part.
1.11260 = Diff. of Off-sets.

A. 51.86222 = Whole Content.
4

R. 3.44888
40

P. 17.9552

	A.	R.	P.
Answer	51 :	3 :	17.9552.

EXAMPLE IV.

What is the content of the Black Common, p. 64.

Off-sets to the Right Hand, or within the Common.

3650
13300
15700
18000
23300
17400
36600
26200
29600

183750 on AB.

1350
910
2500
7200

11960 on BC.

4480
22800
66200
48450
59700
181500
167400
351750
78500
4940

985720 on CD.

18600
2500
46000
172800
38200
50400
59000

387500 on FG.

5200
6000
6850
6855
4225
9800
10850
12600
13800
15300
15350
62250
93750
182700
151600
110100
164700
18250
3640

893820 on EF.

183750 on AB.
11960 on BC.
985720 on CD.
387500 on FG.
893820 on EF.

2462750 = Sum.

Off-sets to the Left, or without.

700
5050
15700
3360

24810 on AB.

3900
11400
41500
39200
1905

97905 on BC.

4100
22800
19800
79500
65550
74250
1200

267200 on CD.

7800
8400

16200 on FG.

24810 on AB.
97905 on BC.
267200 on CD.
16200 on FG.

406115 = Sum.

```
             2)
   on DE = 2184
           ----
           1092
            296 = Off. Perp.
           ----
           6552
          9828
         2184
        -------
         323232 = Off. to R. H.
        2462750 = Former Sum.
        -------
        2785982 = Sum of R. H. Off-sets.
         406115 = Sum of L. H. Off-sets.
        -------
        2379867 = Difference of Off-sets.
```

Right-lined Part found p. 96.

```
is A. 228.18639
       23.79867 = Difference of Off-sets.
      ---------
   A. 251.98506
              4
      ---------
   R.   3.96024
             40
      ---------
   P.   37.6096            A.   R.   P.
               Answer 251 : 3 : 37.6
```

PROB. XIII.

Having two sides and one angle, or two angles and one side, to find the area of a triangle.

RULE.

If the base be given, find the altitude; if not, find both base and altitude by Trigonometry, and then the area by Prob. V.

EXAMPLE I.

What is the content of the triangle ABC in Crow Park, p. 68.

As radius - - - - - -	Log.	10.
To AB = 320 - - - -	L.	2.505150
So is sine of < BAC = 38° 40′	L.	9.795733
To the altitude of Δ ABC = 200	L.	2.300883

Base AC = 630
Half Alt. = 100
Area = 63000

EXAMPLE II.

What is the content of the triangle ABC in Sheep Park, p. 69.

AB = 864
BC = 696

$$
\begin{array}{r}
180^\circ\ 00' \\
< B = \underline{73\quad 30} \\
2)\underline{106\quad 30} \\
53\quad 15
\end{array}
$$

$$
\begin{array}{r}
864 \\
\underline{696}
\end{array}
$$

As $AB + BC = 1560$ - - Log. 3.19312
To $AB - BC = 168$ - - L. 2.22530
So is T. $\frac{< C + < A}{2} = 53^\circ\ 15'$ L. 10.12683
12.35214
To T. $\frac{< C - < A}{2} = 8^\circ\ 18'$ L. 9.15901

$< A = 44\quad 57$
$< C = 61\quad 33$

As radius - - - - - - Log. 10.
To $AB = 864$ - - - - - L. 2.93651
So is sine of $< A = 44^\circ\ 57'$ - L. 9.84910
To the altitude = 610,4 - - L. 2.78562

As sine of $< C = 61^\circ\ 33'$ Log. 9.94410
To $AB = 864$ - - - - L. 2.93651
So is sine of $< B = 73^\circ\ 30'$ - L. 9.98173
12.91825
To the base $AC = 942{,}2$ - L. 2.97414

2)
Base = 942,2

471,1
Alt. 610,4

18844
4711
28266

Area = 287559,44

PROB. XIV.

Having the sides and angle, to find the area of a rhombus or rhomboides.

RULE.

Find the altitude by Trigonometry, and the area by Prob. VI.

EXAMPLE.

What is the area of a rhombus, whose side is 560 L. and angle 67° 00′?

As radius - - - - -	Log.	10.
To the side = 560 - - -	L.	2.748188
So is sine of the angle = 67° 00′	L.	9.964026
To the altitude = 515,5 - -	L.	2.712214

Alt. = 515,5
Base = 560

30930
25775

Area = 288680

PROB. XV.

Having three ſides and included angles, to find th
area of a trapezium.

RULE.

Find the diagonal and altitudes by Trigonome
try, and the area by Prob. VII. For an example, ſe
Prob. XVII. AHGF in Sheep Park.

PROB. XVI.

Having all the ſides and angles, to find the area o
an irregular polygon.

RULE.

Find the diagonals and altitudes that fall upo
them by trigonometry, the areas of the ſeveral triangl
and trapeziums, by Prob. V. and VII. and add the
all together: the ſum is the whole content. For a
example, ſee next Problem, Sheep Park, the whole wo
for right-lined part.

PROB. XVII.

To find the content of any ſurvey by the graph
meter.

RULE.

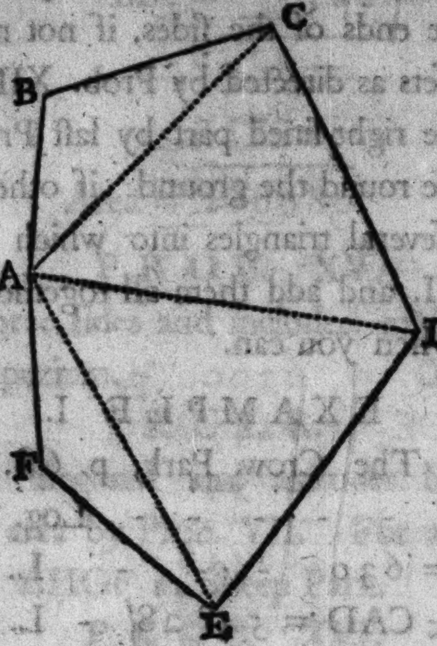
Make a rough draught of it; and join all the points that mark the ends of the ſides, if not meaſured. Do with the off-ſets as directed by Prob. XII. and find the content of the right-lined part by laſt Prob. if the ſurvey was made round the ground: if otherwiſe, find the areas of the ſeveral triangles into which it is reſolved, by Prob. XIII. and add them all together. Apply alſo Prob. VII. when you can.

EXAMPLE I.

The Crow Park, p. 68.

As radius - - - - - -	Log.	10.
To AC = 630 - - -	L.	2.799341
So is S. < CAD = 54° 28′ -	L.	9.910506
To the alt. 512,7 - - -	L.	2.709847
As radius - - - - - -	Log.	10.
To AE = 680 - - - -	L.	2.832509
So is S. < EAD = 51° 45′	L.	9.895045
To the alt. 534 - -	L.	2.727554
As radius - - - - - -	Log.	10.
To AF = 340 - - -	L.	2.531479
So is S. EAF = 24° 30′ -	L.	9.617727
To the alt. of AFE = 141 -	L.	2.149206

The Crow Park, p. 68.



$$\text{Alt.} = \begin{cases} 512{,}7 \\ 534 \end{cases}$$

2)1046,7

523,35

AD Diagon. = 710

52335

366345

ACDE = 371578,5

Alt. = 141
AE Base = 680

1128
846

2)95880

AFE = 47940

ABC = 63000 p. 113.
ACDE = 371578
AFE = 47940

482518

A. 4.82518
4

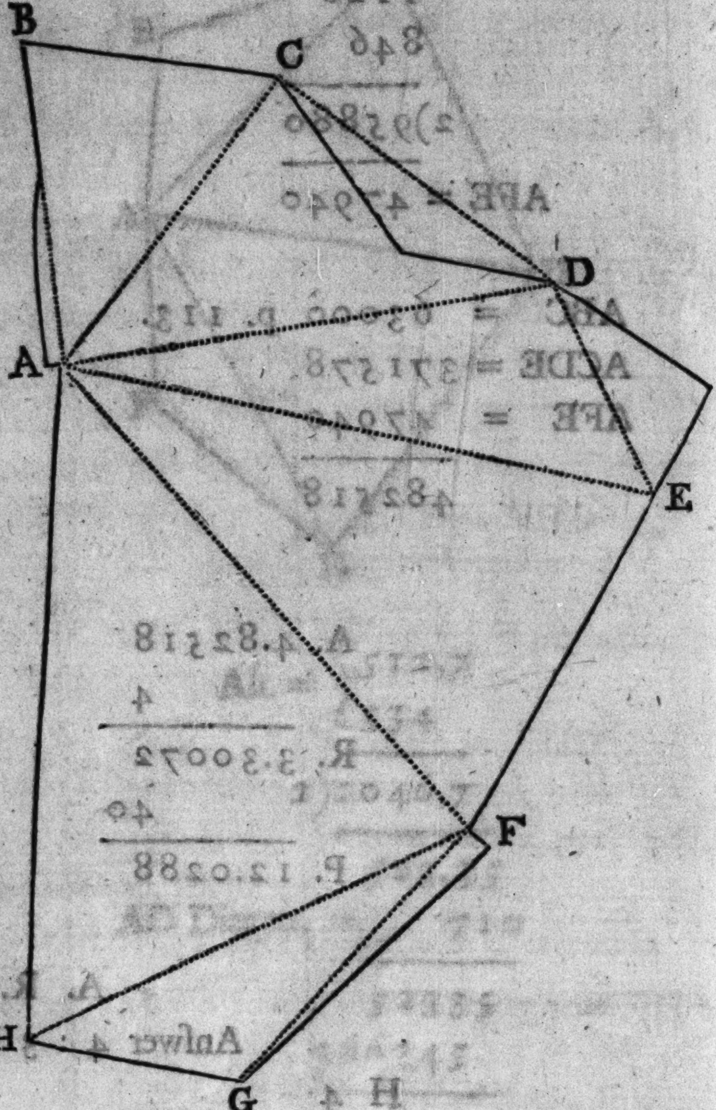
R. 3.30072
40

P. 12.0288

A. R. P.
Answer 4 : 3 : 12

EXAMPLE II.

The Sheep Park, p. 69.



$$180^\circ\ 00'$$
$$\angle G = 120\ \ 50$$
$$2)\ \overline{59\ \ 10}$$
$$\overline{29\ \ 35}$$

$$920$$
$$\underline{570}$$

As $FG + GH = 1490$ - - Log. 3.173186
To $FG - GH = 350$ - - L. 2.544068
So is T. $\frac{GHF + GFH}{2} = 29^\circ 35'$ L. 9.754115

12.298183

To T. $\frac{GHF - GFH}{2} = 7^\circ 36'$ L. 9.124997

$\angle GHF = 37^\circ 11'$
$\angle GFH = 21^\circ 59'$

As S. $\angle GHF = 37^\circ 11'$ - Log. 9.781301
To $GF = 920$ - - - - L. 2.963788
So is S. $\angle G = 120^\circ 50'$ - - L. 9.933822

12.897610

To $HF = 1307$ - - - - L. 3.116309

$\angle AHG = 70^\circ 38'$
$\angle GHF = \underline{37\ \ 11}$
$\angle AHF = 33\ \ 27$

As radius	- - - -	Log.	10.
To GH = 570	- - - -	L.	2.755875
So is S. < GHF = 37° 11′		L.	9.781301
To the altitude = 344,5	-	L.	2.537176

As radius	- - - -	Log.	10.
To AH = 1802	- - -	L.	3.255750
So is S. < AHF = 33° 27′		L.	9.741316
To the altitude = 993,2	-	L.	2.997066

$$\text{Altitude} = \begin{cases} 344,5 \\ 993,2 \end{cases}$$

$$\begin{array}{r} 2)\underline{1337,7} \\ 668,85 \end{array}$$

$$\begin{array}{rr} \text{Diagonal} = & \underline{1307} \\ & 468195 \\ & 200655 \\ & \underline{66885} \\ & 874186,95 = \text{AHGF}. \end{array}$$

$$\begin{array}{lrr} < \text{BCD} = & 149° & 27′ \\ < \text{BCA} = & 61 & 33 \\ \hline < \text{ACD} = & 87 & 54 \\ & 180 & 00 \\ \hline & 2)92 & 06 \\ \hline & 46 & 03. \end{array}$$

p. 113. where is CA = 942,2.

	942,2		
	920,		
As CA + CD = 1862,2	-	Log.	3.269980
To CA − CD = 22,2	-	L.	1.346353
So is Tang. of 46° 03′	- -	L.	10.015921
			11.362274
To Tang. of 0° 42′	- - -	L.	8.092294
< CDA = 46 45			

As radius	- - - - -	Log.	10.
To CD = 920	- - - - -	L.	2.963788
So is S. < CDA = 46° 45′	-	L.	9.862353
To the altitude 670,1	- - -	L.	2.826141

As S. < ADC = 46° 45′	-	Log.	9.862353
To AC = 942,2	- - -	L.	2.974143
So is S. < ACD = 87° 54′	-	L.	9.999708
			12.973851
To AD = 1293	- - - - -	L.	3.111498

< CDE =	152°	20′
< CDA =	46	45
< ADE =	105	35
	180	00
	2)74	25
	37	12

1293
610

As AD + DE = 1903	- -	Log.	3.279439
To DA — DE = 683	-	L.	2.834421
So is tangent of 37° 12′	-	L.	9.880265
			12.714686
To tangent of 15° 15′	- -	L.	9.435247

< AED = 52 27

As S. < AED = 52° 27′	-	Log.	9.899176
To AD = 1293	- - -	L.	3.111498
So is S. < ADE = 105° 35′	-	L.	9.983735
			13.095233
To AE = 1570	- - - -	L.	3.196057
As radius	- - - - - - -	Log.	10.
To DE = 610	- - - -	L.	2.785330
So is S. < AED = 52° 27′	-	L.	9.899176
To the altitude = 483,6	- -	L.	2.684506

< DEF = 123° 28′
< AED = 52 27
< AEF = 71 01
180 00
2) 108 59
54 30

	1570		
	1027		
As AE + EF = 2597	- -	Log.	3.414472
To AE − EF = 543	- -	L.	2.734800
So is tangent of 54° 30′	- -	L.	10.146732
			12.881532
To tangent of 16° 12′	- -	L.	9.467060
< AFE = 70 42			

As S. < AFE = 70° 42′	-	Log.	9.974880
TO AE = 1570	- - - -	L.	3.196057
So is S. < AEF = 71° 01′	-	L.	9.975713
			13.171770
To AF = 1573	- - - -	L.	3.196890

As radius	- - - - - -	Log.	10.
To EF = 1027	- - - -	L.	3.011570
So S. < AFE = 70° 42′	- -	L.	9.974880
To the altitude 969,3	- - -	L.	2.986450

AD = 1293
Alt. = 670,1
1293
9051
7758
2)866439,3
△ ACD = 433219,6

2)
Alt. 483,6
241,8
AE = 1570
16926
12090
2418
△ ADF = 379626

Altitude 969,3
AF = 1573
29079
67851
48465
9693
2)1524708,9
△ AEF = 762354,4

Off-sets within, or to the Right Hand.			Off-set without.
5800	610	19600	920
5600	130	79300	84
3900	79300 on DE.	38640	368
2600	920	137540 = Sum.	736
1400	42	77280	77280 on CD.
300	184	60260 = Diff.	
19600 on AB.	368		
	38640 on FG.		

ABC = 2.87559,4 p. 113.
AHGF = 8.74186,9
ACD = 4.33219,6
ADE = 3.79626
AEF = 7.62354,4

A. 27.36946,3 = Right-lined Part.
60260 = Difference of Off-sets.

A. 27.97206
4

R. 3.88824
40

P. 35.5296

	A.	R.	P.
Anſwer	27 :	3 :	$35\frac{1}{2}$

Theſe Problems, I think, may ſuffice for finding the contents of all ſuch pieces of land as are commonly met with, by whatever method they are ſurveyed. But as there are ſome other figures to be meaſured ſometimes, viz. the Circle and its parts, the Regular Polygon and the Ellipſis; they ſhall be next explained.

Theſe figures may be frequently found in incloſures, tho' ſeldom or never in open fields.

As for the Parabola, Hyperbola, and other curves, if any thing like them ſhould ever appear, the dimenſions may be taken, and contents found, as directed for ſurveys by the chain, with ſufficient exactneſs.

OF THE CIRCLE AND ITS PARTS.

The Circle is the moſt perfect of all figures, and contains more than any other figure whatſoever of an equal circumference. It is likewiſe the moſt uſeful figure, for by the parts of its periphery, viz. Arches, Degrees and Minutes, &c. and by the right lines drawn within it, and about it, as ſines, tangents &c. it gives rules for meaſuring almoſt every thing, and for making meaſuring inſtruments &c. But this perfect and uſeful figure cannot be meaſured itſelf exactly; tho' many have attempted to find out ſome exact method for finding its content: the moſt ſucceſsful have been ARCHIMEDES about 2000 years ago, METIUS and VAN CULEN lately. Their proportions for finding the circumference by the diameter are,

ARCHIMEDES, As 7 to 22, ſo is the diameter to the circumference. This is exact for any number under 1000.

METIUS, As 113 to 355. Exact to one million.

VAN CULEN, As 1 to 3.14159265358979323846264338327950288. Exact to One million of millions of millions of millions of millions.

This proportion of VAN CULEN's has been so highly esteemed, that it is engraved on his monument at Leyden in Holland. Of this large number, the first six places, 3.14159, or by raising the fifth, 3.1416, may serve in most cases, where very great exactness is not required, or not to be attained by reason of inaccuracy in measuring the diameter.

PROB. XVIII.

Having the diameter of a circle, to find the circumference.

RULE.

Multiply the diameter by 22, and divide the product by 7, or multiply by 355, and divide by 113; or multiply it by 3.14159 &c.

EXAMPLE.

What is the circumference of a circle, whose diameter is 1120 links?

```
   1120                       355
     22                      1120
   ----                      ----
    224                       710
   224                       355
   -----                    355
 7)24640                    -----
   -----           113)397600(3518,58
   3520                 339
                        ---
                         586
                         565
                         ---
                          210
                          113
                          ---
                           970
                           904
                           ---
                            660
                            565
                            ---
                             95

        3.14159
           1120
        -------
         628318
       314159
      314159
      ----------
      3518.5808

                 Answer 3518,581
```

PROB. XIX.

Having the circumference, to find the diameter.

RULE.

Multiply by 7, and divide by 22; or multiply by 113, and divide by 355; or divide the circumference by 3.1416.

EXAMPLE.

What is the diameter of a circle, whose circumference is 1000000?

```
3,1416)1000000(318309
        94248
        -----
         57520
         31416
         ------
         261040
         251328
         ------
           97120
           94248
           ------
           287200
           282744
```

PROB. XX.

Having diameter and circumference, to find the area.

RULE.

Multiply half of one by half of the other.

EXAMPLE I.

What is the area of VAN CULEN's circle?

$$
\begin{array}{lll}
 & & 2)\ \\
 & \text{Circumference} = & 3.14159265359 \\
 & & \overline{} \\
 & 2) & 1.570796326795 \\
\text{Diameter} & 1,0 & ,5 \\
 & \overline{} & \overline{} \\
 & ,5 & ,785398163397 5 = \text{Area.}
\end{array}
$$

EXAMPLE II.

What is the area of a circle, whose circumference is 1?

$$
\begin{array}{ll}
 & 2) \\
\text{Diameter} & ,318309 \\
 & \overline{} \\
 & ,1591545 \\
 & \quad\quad ,5 = \tfrac{1}{2}\ \text{Circumference.} \\
 & \overline{} \\
 & ,07957725 = \text{Area.}
\end{array}
$$

These are the necessary Problems concerning the circle, which every Surveyor should know. There are other things relating to it, which shall be explained in the following articles, without the unnecessary formality of rules and examples.

1. If you would find the area, by the diameter alone, multiply the square of it by the area of VAN CULEN's circle, viz. ,785398.

2. If you would find the area by the circumference alone, multiply the square of it by ,079577. For al

circles are to one another as the ſquares of their diameters &c.

3. If you take VAN CULEN's area, and extract the ſquare root of it, that root will be a multiplier for the diameter, to find the ſide of a ſquare equal to the circle. And if you take the area of a circle whoſe circumference is 1, and extract the ſquare root of it, that will be a multiplier for the circumference to the ſame purpoſe.

4. If you extract the ſquare root of half the ſquare of the diameter, that root will be the ſide of the inſcribed ſquare. And if you extract the ſquare root of half the ſquare of ,318309, that will be a multiplier for the circumference, to the ſame purpoſe: and if you multiply VAN CULEN's circumference by the ſame root, the product will be another multiplier for the diameter to find the ſide of the inſcribed ſquare.

5. The diameter is the ſide of the circumſcribing ſquare, or diagonal of the inſcribed ſquare.

6. You may find the diameter of the circumſcribing circle for any ſquare, by extracting the ſquare root of double the area of it; or the ſquare root of 2 will be a multiplier for the ſide of the ſquare to the ſame purpoſe.

7. Such queſtions as ſuppoſe the area of the circle given, belonging properly to the third part, need not be mentioned here.

8. The proportion of the circle to the circumſcribing ſquare has been the occaſion of the following fanciful queſtion being ſometimes propoſed, with which I ſhall conclude what I have to ſay, in this place, about the circle.

A landed man two daughters had,
And both were very fair:
He gave to each a piece of land,
One round, the other ſquare:
At twenty pounds an acre juſt,
Each piece its value had;
The ſhillings that encompaſs each
The price exactly paid.
If croſs the ſhilling be an inch,
And it is very near:
How much above the circle is
The exceſs of the ſquare?

To work this queſtion. Divide four times the number of ſquare inches in an acre by the number of ſhillings in the price: the quotient is the ſide of the ſquare and diameter of the circle required. See the work.

Inches in one link 7,92 £.
7,92 20
20
1584
7128 Price = 400
5544

Sq. In. in a Sq. L. 62,7264
100000

Square In. in an A. 6272640
4

4|00)250905|60

Side of the ſquare = 62726,4 Inches or ſh.
4

Circumference = 250905,6

62726,4, ſquared and divided by 6272640, will give 627 acres and ,264.

A.
627.264 = Area of the ſquare.
20

£. 12545,28 = Value of the ſquare.
20

250905,6 ſh. = the circumference.

Diameter of the circle 62726,4
3,1416

3763584
627264
2509056
627264
1881792

Circumference = 2)197061,25824

98530,62912

$\frac{1}{2}$ Circumference = 98530,62912
$\frac{1}{2}$ Diameter = 31363,2

19706125824
29559188736
59118377472
29559188736
9853062912
29559188736

Area = 3090235827,216384 in Sq. inches.

A.

```
627264|0)309023582|7,216384(492.6531456 = Area in acres &c.
         2509056 ········          20
         -------          -----------------
          5811798        £. 9853,062912 = Value of the circle.
          5645376                    20
          -------          -----------------
          1664222         197061,25824 sh. = Circumference.
          1254528
          -------
           4096947
           3763584
           -------
            3333632
            3136320
            -------
             1973121
             1881792
             -------
              913296
              627264
              -------
              2860323
              2509056
              -------
               3512678
               3136320
               -------
                3763584
```

	Contents. A.	Values. £.
Square	= 627,264	12545,28
Circle	= 492,653	9853,06
Diff.	A. 134,610	£. 2692,21

Thus the ſquare is the better portion by 134 $\frac{1}{2}$ acres, and 2692 £.

I proceed now to the parts of the Circle, which are the Sector, Segment and Annulus. The area of any ſector an aliquot part of the whole circle is found by dividing the whole area by the meaſure of that part; as 2 for the ſemicircle, 4 for the quadrant, 6 for the ſextant, 8 for the octant &c. but as it is not always diſcoverable at firſt ſight, without taking the angle, what part of the whole, any ſector that is to be meaſured may be, you may take the following rules for finding the content of any ſector whatſoever.

PROB. XXI.

Having the radius and angle of any ſector, to find the arch line.

RULE.

Double the radius for the diameter, and find the whole circumference: then if the ſector be leſs than a ſemicircle, multiply the whole circumference by the meaſure of the angle in degrees and decimals, and divide the product by 360, the quotient is the arch line: but if it be greater than a ſemicircle, ſubtract that meaſure from 360° 00′, and work with the remainder.

EXAMPLE I.

What is the arch line of a ſector leſs than a ſemicircle, whoſe radius is 48 yards, and angle 56° 15′?

Radius =	48	3,1416
	2	96
Diameter =	96	188496
		282744
	Circumference =	301,5936

6|0)15′,00

,25 = Decimal of 15 Minutes.

```
    301,5936
       56,25
  ----------
    15079680
   6031872
  18095616
 15079680
 -------------
 16964,640000
```

```
36|0)1696|4,64(47,124
     144
     ---
      256
      252
      ---
       44
       36
       --
       86
       72
       ---
       144
```

Anſwer 47,124 Yards.

EXAMPLE II.

What is the arch line of a ſector, greater than a ſemicircle, whoſe radius is 72 links, and angle 78° 45′?

```
  72            3,1416
   2               144
 ---           -------
 144            125664
               125664
               31416
               --------
 360° 00′      452,3904
  78,75
 ------
 281,25
```

```
    452,39      36|0)12723|4,6875(35343,
    281,25           108
   --------         -----
    226195           192
   90478             180
  45239             -----
 361912              123
 90478               108
----------          -----
127234,6875          154
                     144
                    -----
                     106
```

Answer 353,43 Links.

PROB. XXII.

Having the radius and arch line, to find the area of any ſector.

RULE.

Multiply the one by half of the other.

EXAMPLE I.

What is the area of a ſector, whoſe radius is 48 yards, and arch 47,124 yards?

```
          47,124
              24
        ————————
          188496
          94248
        ——————————— P.    Y.
30,25)1130,976(37 -- 11,726
          9075
        ————————
          22347
          21175
        ————————
           11,72
```

Anſwer 37 Poles, $11\frac{3}{4}$ Yards.

EXAMPLE II.

What is the area of a ſector, whoſe radius is 72 links, and arch 353,43 links?

```
   353,43          A. 0.12723,48
       36                     4
  ————————          ————————————
   212058          R. 0.50893
  106029                      4
  ————————          ————————————
  12723,48         P. 20.3572
```

Anſwer 20.3572 Poles.

PROB. XXIII.

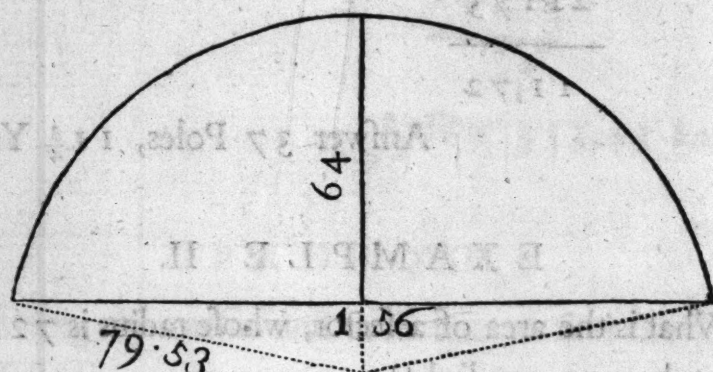
Having the chord and versed ſine (or height) of any ſegment, to find the diameter of the whole circle.

RULE.

Divide the ſquare of half the chord by the height, and to the quotient add the height.

EXAMPLE I.

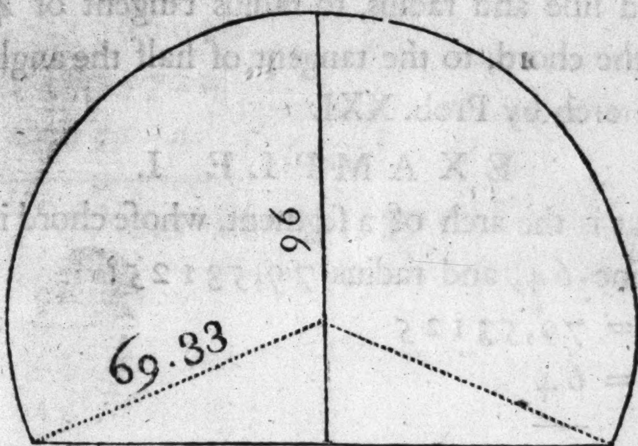
What is the diameter of the whole circle, a ſegment whereof has a chord of 156 links, and verſed ſine 64 links?



2)156	64)6084(95,0625	
78	576	64
78	324	159,0625 = Diameter.
624	320	
546	400	2)159,0625
Sq. 6084	384	79,53125 = Rad.
	160	
	128	
	320	

EXAMPLE II.

What is the radius of the circle, a ſegment whereof has a chord of 128, and verſed ſine 96?



128

2)128
64
64
———
256
384
———
4096

96)4096(42,666
384
———
256
192
———
640
576
———
64

42,666
96
———
2)138,666
———
Radius = 69,333

PROB. XXIV.

Having the chord, verſed ſine and radius, to find the arch line of a ſegment.

RULE.

Find the angle of the ſector containing, or contained within, the ſegment, by this proportion, as the difference of verſed ſine and radius, to radius tangent of 45°, ſo is half the chord, to the tangent of half the angle: then find the arch by Prob. XXI.

EXAMPLE I.

What is the arch of a ſegment, whoſe chord is 156, verſed ſine 64, and radius 79,53125?

Radius = 79,53125
V. ſine = 64

As Diff. 15,53125	- -	Log.	1.191206
To radius, tangent of 45° 00′	-	L.	10.
So is half the chord = 78	-	L.	1.892095
To tangent of half the < = 78° 45′		L.	10.700889
2			
Angle of the ſector = 157,30			

3,1416
Diameter = 159

282744
157080
31416

499,5144

```
     499,5144
        157,5
     --------
     24975720
    34966008
   24975720
   4995144
   --------
36|0)7867|3,518(218,5375
  72
  --
   66
   36
   --
   307
   288
   ---
    193
    180
    ---
     135
     108
     ---
      271
      252
      ---
       19
```

Thus the arch of this segment is found 218,538.

EXAMPLE II.

What is the arch of a segment, whose chord is 128, versed sine 96, and radius 69,33333?

Versed sine	= 96		
Radius	= 69,333		
Difference	= 26,666	- Log.	1.425968
To radius	- - - - -	L.	10.
So is 64	- - - - -	L.	1.806180
To tangent of	67° 23′	- L.	10.380212
	2		
Angle of the sector	= 134 46		

Diameter = 138,666
3,1416

832000
138666
554666
138666
416000

435,6152666

360,00 435,6152
184,75 225,25

225,25 21780760
8712304
21780760
8712304
8712304

36|0)9812|2,3238(272,562
72

261
252

92
72

202
180

223
216

72 Answer 272,562

PROB. XXV.

Having chord, versed sine, radius and arch, to find the area of any segment.

RULE.

Multiply the arch by half the radius, and the chord by half the difference of versed sine and radius; if the radius is greatest, subtract the last product from the first, the remainder is the area: but if the versed sine be greatest, add the two products together, and the sum is the area required.

EXAMPLE I.

What is the area of a segment, whose chord is 156, versed sine 64, radius 79,53125 and arch 218,538?

Radius = 79,53125
Half the arch = 109,269

```
      71578125
     47718750
    15906250
   71578125
  7953125
  -------------
  8690,30015625
```

Diff. of V. S. and Rad. = 15,53125
Half the Chord = 78

```
  12425000
 10871875
 ---------
 1211,4375
```

Second Product = 1211,4375
First Product = 8690,3001
Area of the segment = 7478,8626 Answer 7479

EXAMPLE II.

What is the area of a ſegment, whoſe chord is 128, verſed ſine 96, radius $69\frac{1}{3}$, and arch 272,562?

Half the Arch =	136,281	Chord =	128
Radius =	$69\frac{1}{3}$	Half Diff. =	$13\frac{1}{3}$
	1226529		384
	817686		128
	45427		$42\frac{2}{3}$
	9448,816		$1706\frac{2}{3}$

First Product = 9448,816
Laſt Product = 1706,666

Area of the ſegment = 1115,5482

PROB. XXVI.

Having the breadth and inſide diameter, to find the area of an annulus, or circular ring.

RULE.

To the inſide diameter add double the breadth, and multiply the ſquare of the ſum by ,7854: from this product ſubtract the product of the ſquare of the inſide diameter by the ſame multiplier: the remainder is the area required.

EXAMPLE.

What is the area of a ring 50 links broad, whoſe inſide diameter is 90 L?

50
2
100
90
190
190
171
19
36100

,7854
36100
7854
47124
23562
28352,94

,7854
Square of 90 = 8100
7854
62832
6361,74

28352,94
6361,74
21991,2 = Area.

PROB. XXVII.

To find the area of a mix'd-lined figure, that is, bounded with right lines and arches.

RULE.

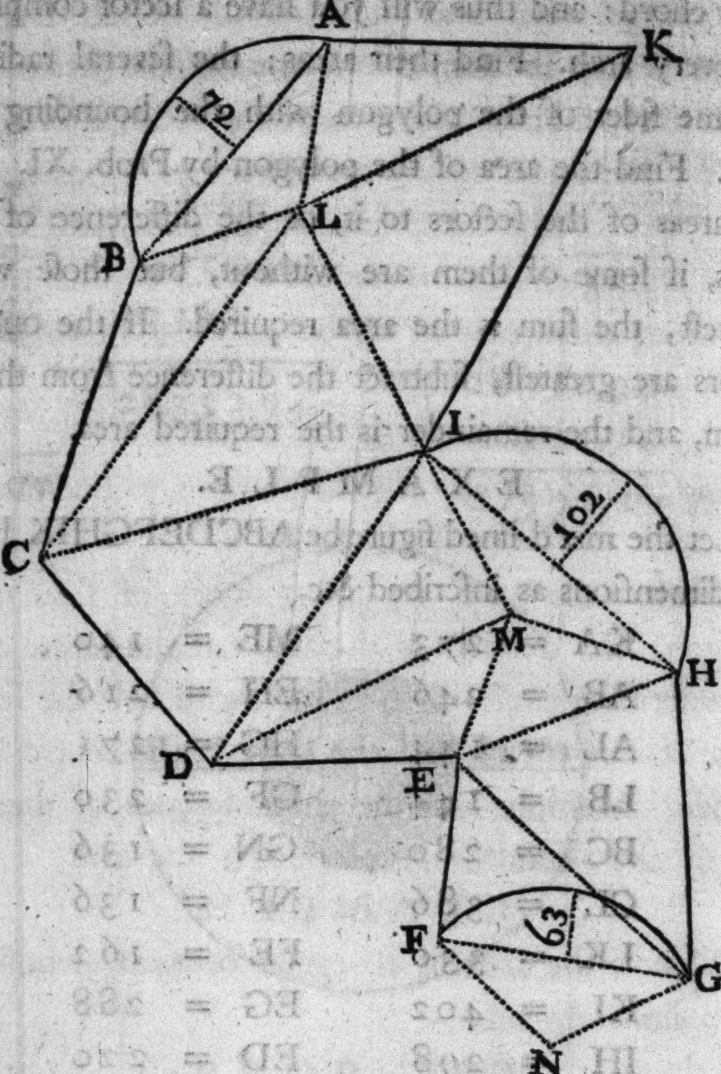
Draw the chords of the arches and versed sines: find the diameters and centres: draw a radius to each end of

every chord: and thus will you have a ſector compleated for every arch. Find their areas; the ſeveral radii will become ſides of the polygon with the bounding right lines. Find the area of the polygon by Prob. XI. Add the areas of the ſectors to it, or the difference of their areas, if ſome of them are without, but thoſe within greateſt, the ſum is the area required. If the outward ſectors are greateſt, ſubtract the difference from the polygon, and the remainder is the required area.

EXAMPLE.

Let the mix'd-lined figure be ABCDEFGHIK having the dimenſions as inſcribed &c.

KA	=	273	ME	=	140
AB	=	246	EH	=	216
AL	=	144	HG	=	271
LB	=	144	GF	=	230
BC	=	280	GN	=	136
CL	=	386	NF	=	136
LK	=	330	FE	=	162
KI	=	402	EG	=	288
IH	=	298	ED	=	220
HM	=	160	DC	=	234
MI	=	160	CI	=	354
ID	=	340	IL	=	236
DM	=	300			



The above mix'd-lined figure you ſee is reduced by the radii of the ſectors to the irregular polygon LBCDEFNGHMIKAL, and the two ſectors within ALB and IMH. You may find the area of the poly-

gon by Prob. XI. and those of the sectors by Prob. XXII. From the sum of the areas of the sectors ALB and IMH, subtract the area of the sector FNG, and add the remainder to the area of the polygon: the sum is the area of the mix'd-lined figure ABCDEFGHIK. To insert the whole work should seem a needless trouble, as nothing either new or further explanatory can be seen in it.

PROB. XXVIII.

Having the side of any regular polygon, to find the area.

RULE.

Divide 360° 00′ by the number of sides of the polygon; the quotient is the angle at the centre of the circumscribing circle: then as radius, to half the side, so is the co-tangent of half the angle at the centre, to the altitude. Multiply half the circumference of the polygon by the altitude; the product is the area.

EXAMPLE.

What is the area of a regular octagon, whose side is 60 links?

8)360° 00′ — 2)45° 00′

45 00 = Angle at the centre $\frac{1}{2}$ = 22 30

90° 00′
22 30

67 30 = Comp. of < at the centre.

As radius	- - - - - -	Log.	10.
To half the side = 30	- -	L.	1.477121
So is tangent of 67° 30′	-	L.	10.382776
To the altitude = 72,42	- -	L.	1.859897

$$\begin{array}{rl} 60 & \text{Altitude} = 72{,}42 \\ 8 & \tfrac{1}{2}\ \text{Circum.} = \quad 240 \\ \hline 480 = & \text{Circum.}\quad 28968 \\ & \qquad 14484 \\ \hline & \qquad 17380{,}8 = \text{Area.} \end{array}$$

This may be done also by the help of a table, which you shall have in the Third Part, where the principal use of it is.

PROB. XXIX.

Having the greatest length and greatest breadth of an ellipsis, to find its area.

RULE.

Multiply the length by the breadth, and the product by ,785398.

EXAMPLE.

What is the area of an ellipsis, whose greatest length is 517, and breadth 389?

Breadth	389	,785398
Length	517	201113
	2723	2356194
	389	785398
	1945	785398
Product =	201113	785398
		1570796
		157953,747974 = Area.

The particular explication and construction of this figure will come in more properly in the Third Part, where it is necessary to be fully understood. I shall therefore say no more of it here.

As the First Part concluded with finding the diameter of the earth, this shall end with finding its superficial content.

PROB. XXX.

Having the diameter, to find the superficial content of any globe.

RULE.

Find the circumference, and multiply it by the diameter. Or multiply the square of the diameter by 3.14159265.

EXAMPLE.

How many acres in the surface of the terraqueous globe, supposing the diameter to be 7967,7 English miles?

Diameter = 7967,7	3.14159265
7967,7	63484243,3
557739	942477795
557739	942477795
478062	1256637060
717093	628318530
557739	1256637060
Sq. = 63484243,29	2513274120
	1256637060
	942477795
	1884955590
	Area = 199441632,142091745

199441632 Square miles.
640
797766528
1196649792
127642644480 Acres.

Answer 127642644480

PART III.

LAYING OUT &c.

WHEN it is required to form one piece of ground of any figure convenient or proposed equal to another piece of any other figure; or to exchange one piece for another of equal content, or unequal in a given proportion, the given piece being first surveyed, and its content found: this is what we call Laying Out Ground, or tracing out, upon the ground, any figure that shall contain any number of acres &c. required.

As I have, in the Second Part, rejected the method of finding contents by measures taken from draughts how exactly soever made, and instead thereof shewn methods far more exact, and consequently far better, so in this and the next Part, I propose to give you only such rules as can be easily and surely applied in real practice, and proven to the satisfaction of all concerned; not so much as mentioning the geometrical methods published in several books; because none of them are very sure nor easy either to apply or prove, and some of them are altogether impracticable in the field. For tho' a scale and compasses can do many pretty things upon paper, and surprize you with a curious construction, and a number of right lines and arches; and tho' the chain can

serve for both within its own length; yet you will find there is some difference betwixt working upon paper and upon the ground, and the larger the field, the greater is the difficulty and uncertainty. But what is to be done, when your geometrical lines must sometimes end in a place where the chain cannot run, as in a thicket, marsh &c. how is it to sweep an arch there? And tho' you can use a long measuring line, where the extent required exceeds the length of the chain; no measuring line can be so exact, and a small error in a radius, and the arch it sweeps, makes a sensible error in the whole work: consequently the more of these radii and arches, the worse. I shall therefore, both in laying out, and dividing ground, always suppose contents given, because they can and ought always to be found, for satisfaction in the proof of the work, if there was no other reason.

PROB. I.

To reduce acres, roods &c. to square links or yards &c.

RULE.

If the content is expressed in acres, roods, poles and decimals; divide the poles and decimals by 40, and to the quotient prefix the roods, divide it by 4, and to the quotient prefix the acres, annexing cyphers, if needful, to make up five decimal places; then make them all integers. Or, multiply the acres by 4, and take in the roods, the product by 40, taking in the poles and

this product by 30,25 for yards, by 36 for ells, or by 625 for links.

EXAMPLE.

How many square links &c. in 23 acres, 1 rood, 19,52 poles?

A. R. P.
23 : 1 : 19,52

4|0)1|9,52
4)1,488
23,37200

23
4
93
40
P. 373952

3739,52
625
1869760
747904
2243712
2337200 Links.

P.
3739,52
$30\frac{1}{4}$
112185,6
934,88
113120,48 Yards.

F.
3739,52
36
2243712
1121856
134622,72 Ells.

Answer 2337200 square links, or 113120,48 yards, or 134622,72 ells.

If it were required to reduce yards to feet, multiply by 9. Ells by 9,5. Links English by ,4356. Links Scots by ,5476.

PROB. II.

To mark, or trace out any line upon the ground.

RULE.

If it be a right line, provide yourself with a sufficient number of stakes 3 or 4 feet long, and sharpened at one end: measure the line by Prob. I. II. or III. of Part I. and as you go on, drive a stake into the ground at the end of every two chains, one, or half a chain, so as they may be all visible in one right line all the way, over the rising and thro' the hollow ground, if there be any; drive a stake at each of the ends, and you have done. If it be an arch, or circumference of a circle or oval, take a sharp iron stake, and with it make a rut in the ground at the end of the radii as you stretch them round about the centre or foci.

PROB. III.

Thro' a given point, to run a right line parallel to a given right line.

RULE.

Set a pole in the point, if not otherwise conspicuous; stake the line as above, and produce it both ways, if need be: observe what angle is formed at any of the stakes with the line and a right line to the point: then

at the point make the ſame angle with the ſame right line, and produce the line that forms this laſt angle as far as needful both ways. This being the ſame with the 31. of 1. of EUCLID, needs no example. The angle is made by turning the diameter to one pole and the index, upon the meaſure, to the other.

PROB. IV.

To prepare the content, when there are off-ſets upon the baſe, or given ſides.

RULE.

If the right hand ſum is greateſt, ſubtract the difference from the given content, and lay out the remainder: if leaſt, add &c. Examples will follow.

PROB. V.

To lay out any quantity of ground propoſed, in a ſquare.

RULE.

Extract the ſquare root of the propoſed area (reduced by Prob. I.) Meaſure a right line (by Prob. II.) equal to this root: raiſe a perpendicular from each end, equal to it, and join the tops of theſe: ſtake all theſe lines, and you have done.

EXAMPLE.

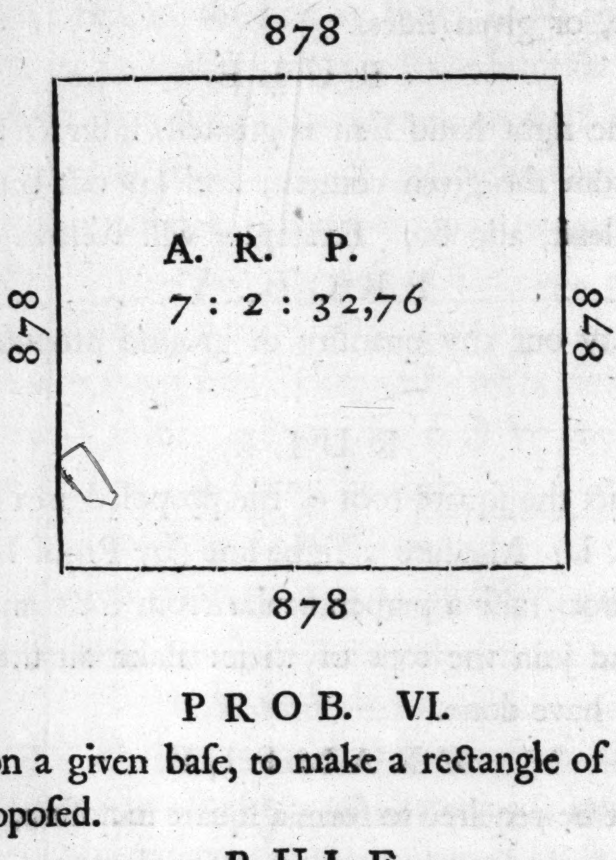
Let it be required to form a ſquare incloſure, containing 7 acres, 2 roods, and 32,76 poles?

A. R. P.   Area = 770475(877,7 = Side.
7 : 2 : 32,76   64

167)1304
1169

4|0)3|2,76
4)2,819
7,70475

1747)13575
12229
1346



PROB. VI.

Upon a given base, to make a rectangle of any content proposed.

RULE.

Divide the content by the base, from each end raise

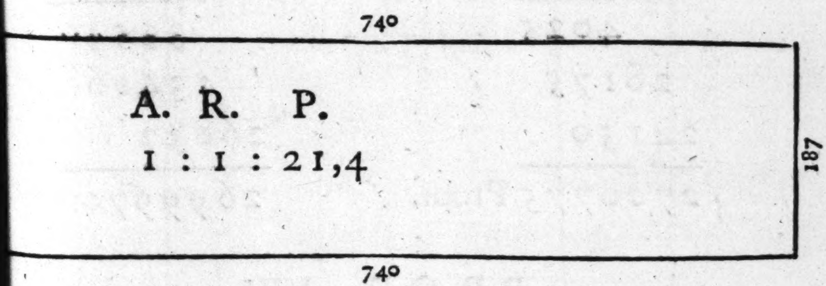
perpendicular equal to the quotient; and join their tops ith a right line, which muſt be equal to the baſe.

EXAMPLE.

Let it be required to lay out 1 acre, 1 rood, and 1,4 poles, in a rectangle whoſe baſe is 740 links?

Area.

```
4|0)2|1,4          74|0)13837|5(187
  4)1,535               74
    1,38375             643
                        592
                         517
                         518
```



PROB. VII.

To make a rectangle of any required content, whoſe gth is any multiple of its breadth.

RULE.

Divide the area by the meaſure of the multiple, and ract the ſquare root of the quotient, this root is the eadth: then make the rectangle by laſt Problem.

EXAMPLE.

Let it be required to lay out 27 acres in a rectang[le] 6 times as long as it is broad?

```
6)2700000
  ---------
   450000(670,8 = Breadth.
   36           6
   ---        -----
127)900       4025 = Length.
    889
    -----------
1340)110000

  4025              670,8
   671               4025
  -----            --------
  4025              33540
 28175             13416
24150             26832
--------          ---------
2700775 Proof.    2699970,0
```

PROB. VIII.

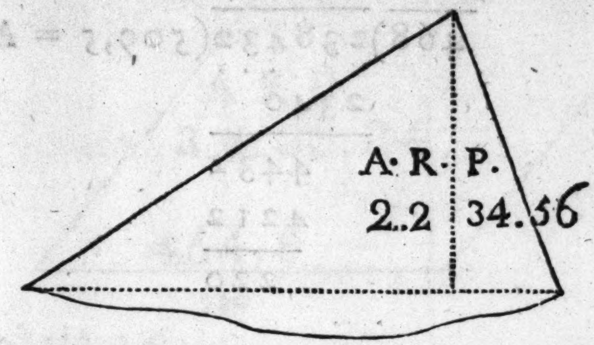
Upon a given base, to make a triangle of any co[n]tent proposed.

RULE.

Divide the area by half the base; from any po[int] of it, raise a perpendicular equal to the quotient; j[oin] the top of it and the two ends of the base.

EXAMPLE.

Let it be required to lay out 2 acres, 2 roods, 34,56 poles in a triangle, whose base is 936 links?



00	00	Off-sets.
100	24	1200
200	32	2800
300	40	3600
400	48	4400
500	56	5200
600	46	5100
700	38	4200
800	30	3400
900	26	2800
36	00	468
		33168

```
      4|0)3|4,56
         4)2,864
           2,71600
  2)936    33168
     468)238432(509,5 = Alt.
            2340
             4432
             4212
              220
```

PROB. IX.

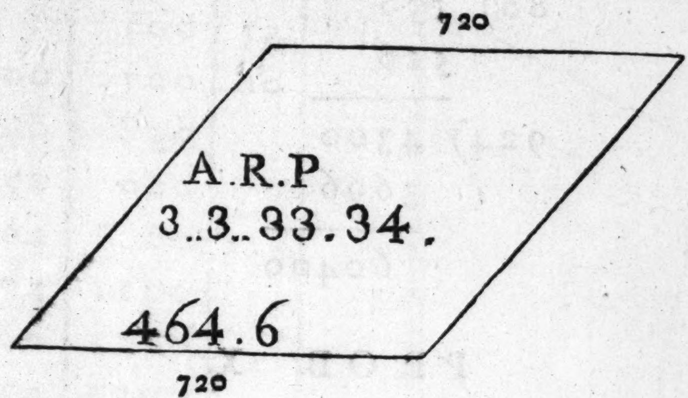
Upon a given baſe, to make a rhombus of any con
tent, leſs than the ſquare of it.

RULE.

Divide the content by the baſe, and ſquare the quo
tient: ſubtract this ſquare from the ſquare of the baſ
and extract the ſquare root of the remainder: lay of
from one end of the baſe, a meaſure equal to this roo
and at the end of it, raiſe a perpendicular equal to th
quotient: join the top of this perpendicular and the en
of the baſe, from which you meaſured: this joining lin
muſt be equal to the baſe: thro' the top of the perpe
dicular, run a parallel to the baſe, equal to it: join th
end of the parallel and the other end of the baſe.

EXAMPLE.

Let 3 acres, 3 roods, 33,34 poles, be laid out in a diamond figure, whose base is 720 links?



4|0)3|3,34

4)3,8335

72|0)39583|7(550 near.

360

358

550
550

275
275

302500

720
720

144
504

518400 Square of the base.
302500 Square of the quotient.

215900 Difference.

$$
\begin{array}{r|l}
 & 215900(464,6 = \text{to be laid off.} \\
 & 16 \\
\hline
86) & 559 \\
 & 516 \\
\hline
924) & 4300 \\
 & 3696 \\
\hline
 & 60400
\end{array}
$$

PROB. X.

Upon a given baſe, to make a rhomboides of any propoſed content.

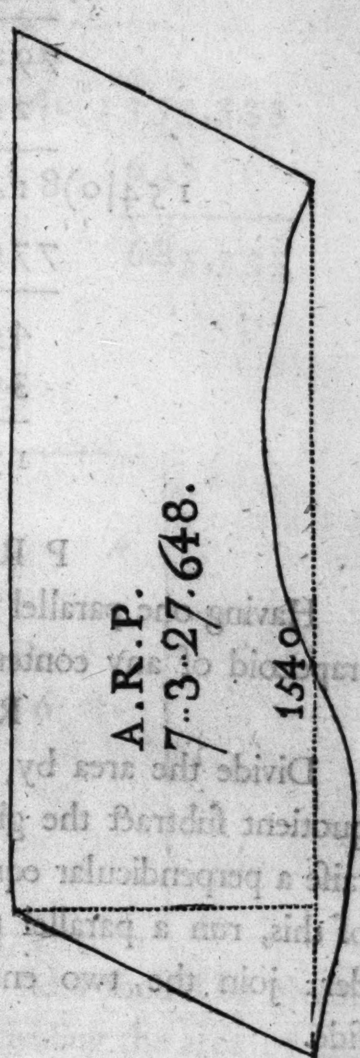
RULE.

Divide the area by the baſe; at any point of it raiſe a perpendicular equal to the quotient; join the top of it and one end of the baſe; from the ſame top, run a right line parallel and equal to the baſe; join the end of this parallel and the other end of the baſe.

EXAMPLE.

Let 7 acres, 3 roods, 27,648 poles, be laid out in a diamond-like figure, the baſe 1540 links?

	00	00
	200	30
	400	36
	500	54
00	700	60
40	800	
52	900	
64	1000	
72	1100	
66	1200	
54	1300	
36	1500	
00	40	



Off-sets.

L. H.	R. H.
2000	3000
4600	6600
5800	4500
6800	5400
6900	
6000	19500
9000	41820
720	22320 = Diff.
41820	

4|0)2|7,648

4)36912

792280

22320

154|0)81460|0(529 = Altitude.

770

446

308

1380

PROB. XI.

Having one parallel ſide and the altitude, to make a trapezoid of any content.

RULE.

Divide the area by half the altitude, and from the quotient ſubtract the given ſide; from any point of it, raiſe a perpendicular equal to the altitude; thro' the top of this, run a parallel to the ſide, equal to the remainder: join the two ends of the parallel and of the ſide.

EXAMPLE.

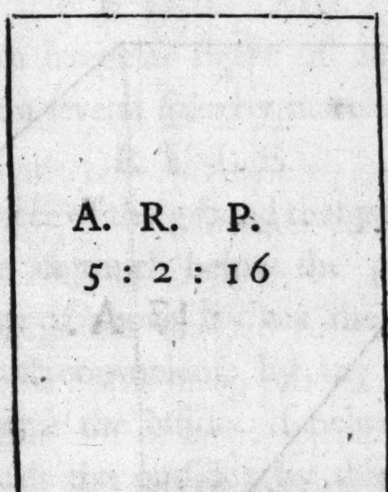
Lay out 5 acres, 2 roods, 16 poles, in a trapezoid whoſe altitude is 840 links, and one parallel ſide 648?

5 A. = 500000.
2 R. = 50000
2)840 16 P. = 10000

420 42|0)56000|0(1333,333
42 648

140 685,333
126

140



Note, If the two parallel ſides ſhould be given, the altitude would be found by dividing the area by half of their ſum.

PROB. XII.

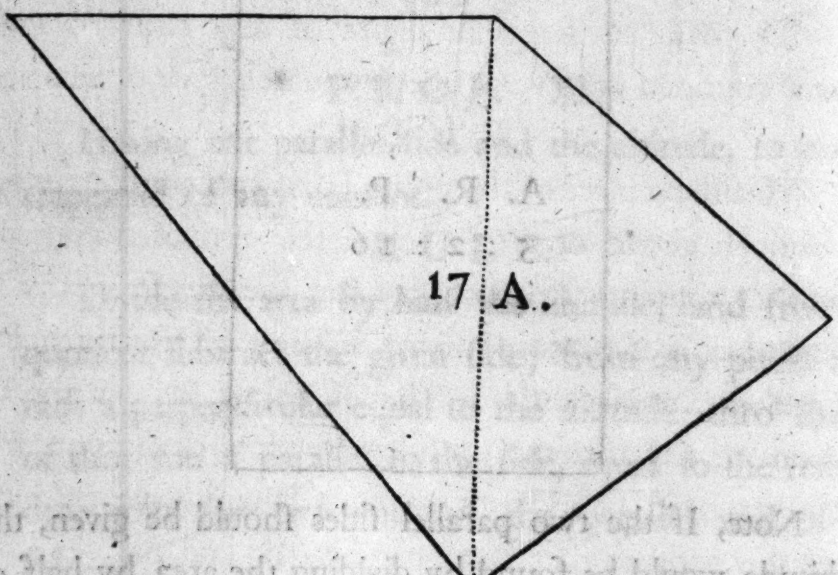
To make a trapezium of any propoſed content, having one ſide, or a baſe given.

RULE.

Lay out one part of the area in a triangle by Prob. VIII. one ſide of which will be a baſe to lay out the remainder upon in another triangle, and the ſame ſide will be the diagonal.

EXAMPLE.

Let 17 acres be laid out in a trapezium, having one ſide 1296 links?



1000000
700000
A. 17.

2)1296
——
648)1000000(1543,2
648
3520
3240
2800
2592
2080
1944
136

2)Alt. & Diag.
771,6)700000(907,2 = Altitude.
69444
55600
54012
1588

PROB. XIII.

To make an irregular figure of any required content, confined on ſeveral ſides by marches, or otherwiſe.

RULE.

Meaſure a piece of the ground that you think may not be much above or much below the propoſed content: find its content: if above, lay out the exceſs, in any figure that is moſt convenient, by any of the foregoing Problems, towards the inſide: if below, lay out the remainder, towards the outſide: by this means you will take from it, or add to it, as much as will leave, or make up the content that is required to be laid out.

I ſuppoſe any example here needleſs.

By theſe Problems any content propoſed may be laid out in any of the figures commonly found, or commonly required in open ground; any piece of ground may be exchanged for another, equal, or leſs, or greater, in any

required proportion; any piece may be added to, or taken from another &c. Let us now see what may be done with other figures.

PROB. XIV.

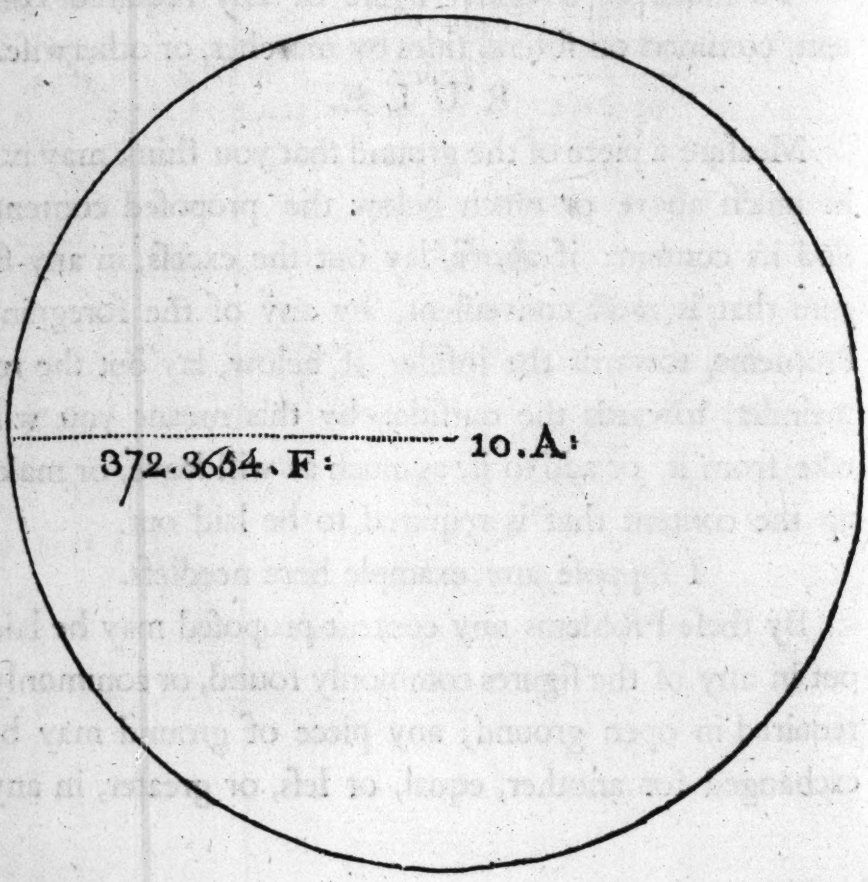
To lay out any quantity of ground in a circle.

RULE.

Divide the area by ,785398, and extract the square root of the quotient for the diameter, the middle point of which is the centre.

EXAMPLE.

What length of a line will trace out a circle of 10 acres content?



```
                                              2)
                   . . . . . .                .
,785398)1000000,000000(1273239,8096(1128,38 =Diameter.
        785398 ......  ——         ————————
        ——————       21)27      Links 564,19 = Radius.
        2146020         21                ,66
        1570796         ———        ————————
        ———————      222)632        338514
         5752240        444        338514
         5497786        ———————    ————————
         ———————    2248)18839    3)372,3654 Feet.
          2544540      17984        ————————
          2356194       ————      124,1218 Yards.
          ———————   22563)85580
           1883460      67689
           1570796      ——————
           ———————      1789196
            3126640
            2356194
            ———————
             7704460
             7068582
             ———————
              6358780
              6283184
              ———————
                75596
```

Anſwer 564,19 links, or 372,3654 feet, or 124,1218 yards.

Proof.

Diameter = 1128,38
3,14159

1015542
564190
112838
451352
112838
338514

Circumference = 2)3544,9073242

Half Circumf. = 1772,4536621
Radius = 564,19

159520829589
177245366621
708981464847
1063472197726
886226831055

Area = 1000000,631620199

PROB. XV.

To find the circumference of a circle of a given content.

RULE.

Divide the content by ,079577, and extract the square root of the quotient.

EXAMPLE.

What is the price of $7\frac{1}{2}$ acres of garden ground, at the rate of as many shillings (or inches) as will ly around it, supposing it a true circle?

Area of 1 acre = 6272640 square inches. See p. 137.

```
                    6272640
                        7,5
               ------------
                    3136320
                    4390848
               ------------                                 2|0) sh.
,079577)47044800,000000(591184373(2431|4,28= Circumf.
        397886 . . . . . . . .     4
        ------                  ----
         725620              44)191            £. 1215,714
         716195                 176                     20
         ------                -----            ---------
           94250            483)1518              s. 14.28
           79577                1449                    12
           ------              ------            --------
          146720           4861)6943              d. 3.36
           79577                4861
           ------               -----
           671420         48624)208273
           636618               194496
           ------               ------
            348020       486282)1377700
            318308               972564
            ------               ------
             297110              405136
             238731
             ------
             583780
             557040
             ------
             267400
```

Proof.

```
                          2)
3.14159)24314,28000(7739,5    = Diameter.
        2199113     3869,75
        -------
         2323150
         2199113          2)24314,28
         -------            --------
         1240370            12157,14
          942477             3869,75
         -------            --------
         2978930             6078570
         2827431            8509998
         -------           10941426
          151499           7294284
                          9725712
                         3647142
                         -------------
                 Area = 47045092,5150=Proof.
```

PROB. XVI.

To make a ſector, an aliquot part of the circle, of any content.

RULE.

Divide 360° 00′ by the meaſure of the part, the quotient is the angle of the ſector: multiply the area by the ſame meaſure, the product is the area of the circle: then find the radius by Prob. XIV.

EXAMPLE.

Let it be required to make a sextant containing 3 roods?

$$6)\underline{360^\circ\ 00'} \qquad \qquad \begin{array}{r} 3 \\ \underline{6} \end{array}$$

$$60\ \ 00 = \text{Angle of the sextant.} \qquad 18\ \text{R.}$$

A. R.

18 R. = 4 : 2 = 450000 square links = Area of the circle.

The rest of the work need not be inserted.

If it should be required to make a sector, that is, any fraction, as $\frac{3}{4}$ of a circle; for example, multiply $360^\circ\ 00'$ by 3, and divide the product by 4 for the angle: and for the radius, multiply the area by 4, and divide by 3 &c. Or, multiply $360^\circ\ 00'$ by the numerator, and divide the product by the denominator for the angle, and multiply the area of the sector by the denominator, and divide the product by the numerator for the area of the whole circle.

PROB. XVII.

Having the inside diameter, to make an annulus, or circular ring, of any content.

RULE.

Multiply the inside diameter by itself, and the product by ,785398. Add this last product to the area

of the ring, the ſum is the area of the whole circle: find the radius by Prob. XIV. and draw the inſide and outſide circumferences.

EXAMPLE.

Let 21991,2 ſquare links be laid out in an annulus, whoſe inſide diameter is 90 links? See Prob. XXVI. of Part II.

PROB. XVIII.

Having the ſide, to make a regular polygon.

RULE I.

Find the angle at the centre by dividing $360^\circ\ 00'$ by the number of ſides: then, as ſine of half the angle at the centre, to ſine of $90^\circ\ 00'$, ſo is half the ſide to the radius of the circumſcribing circle. Draw the circumference of a circle with this radius, and as many chords equal to the given ſide as the number of them requires.

EXAMPLE I.

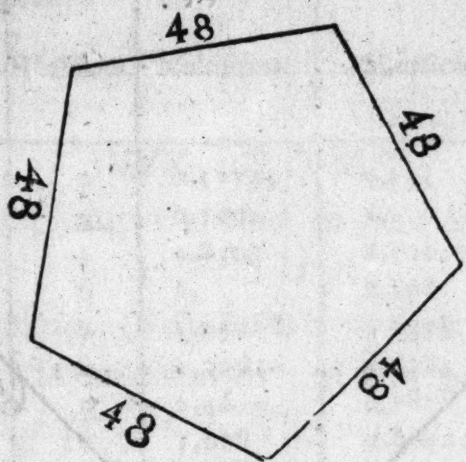
Let it be required to trace out a regular pentagon with a ſide of 48 yards?

2)
Side 48
———
24

Number of ſides $5)360^\circ\ 00'$
———
$2)72\ \ 00$
———
Half angle at the centre $= 36\ \ 00$

As sine of 36° 00′ - - Log. 9.769219
To sine of 90 00 - - L. 10.
So is 24 - - - - - - - L. 1.380211

To the radius = 40,83 - - L. 1.610992



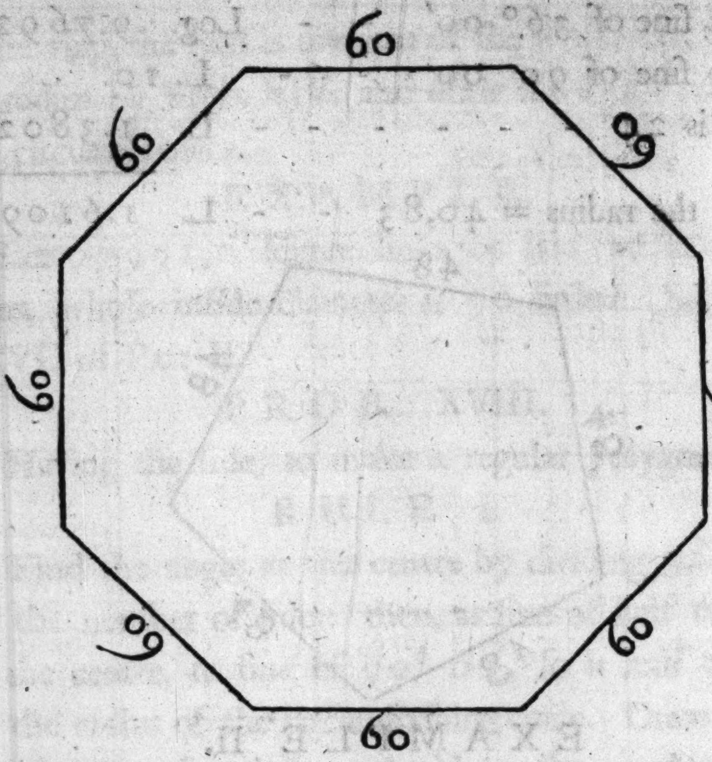
EXAMPLE II.

Let a regular octagon be formed with a side of 60 yards?

			2)
Number of sides	8)360° 00′		Side 60
	2)45 00		30
Half the angle at the centre =	22 30		

As sine of 22° 30′ - - Log. 9.582840
To 30 - - - - - - - L. 1.477121
So is sine of 90° 00′ - - L. 10.

To the radius = 78,39 - - L. 1.894281



Any regular polygon having no more than 12 sides may be described, and the area of it found by the following Tables, thus; multiply the side by the number answering its name in Table I. the product is the radius: multiply the square of the side by the tabular number for its name in Table II. and the product is the area. The multipliers in Table I. are found by this Problem, and these in Table II. by Prob. XXVIII. of Part II. supposing the side 1.

TABLE I. To find the Radius for any Regular Polygon under 13 Sides.			TABLE II. To find the Area of any Regular Polygon under 13 Sides.	
Names.	N° of sides.	Multipliers.	Multipliers.	Names.
Trigon	3	0,57735	0,433	Trigon
Tetragon	4	0,7071	1,	Tetragon
Pentagon	5	0,8507	1,720475	Pentagon
Hexagon	6	1,	2,598	Hexagon
Heptagon	7	1,15228	3,6335	Heptagon
Octagon	8	1,30656	4,8284	Octagon
Enneagon	9	1,4619	6,1818	Enneagon
Decagon	10	1,618	7,6942	Decagon
Endecagon	11	1,7744	9,3638	Endecagon
Dodecagon	12	1,9318	11,196	Dodecagon

EXAMPLE I.

Tabular number for the radius of the pentagon } ,8507

48 = Side.

68056

34028

Radius as before = 40,8336

Side = 48
48
384
192

Square of the side = 2304
Tabular number for the area 1,720475
2304

6881900
5161425
3440950

Area = 3963,9744

EXAMPLE II.

Multiplier for the radius of the octagon 1,30656
Side = 60

Radius as before = 78,3936

Multiplier for its area

Side =	60	4,8284
	60	3600
Square of side =	3600	289704
		144852

See p. 155. 17382,24 = Area.

This Problem gives the most exact rule for making any regular polygon; but when the side is large, it may be more quickly done by the following.

RULE II.

Divide 360° 00′ by the number of sides, and subtract the quotient from 180°, the remainder is the angle of the polygon. Measure out a right line equal to the side, make this angle with another side at the end of the first, do the same with the third side at the end of the second &c. till you have compleated the polygon.

PROB. XIX.

To lay out any quantity of ground in a regular polygon.

RULE.

Find the area of a polygon, of the same name with that required, the side of which is 1. Divide the proposed area by this area, and extract the square root of the quotient: this root is the side of the polygon required: which you may trace out by last Problem.

EXAMPLE.

Let 17382 square links be laid out in the figure of a regular octagon?

Tabular number = 4,8284 = Area of a regular octagon whose side is 1.

Area
4,8284)17382,0000(3600|60 = Side required, as in last Example.
144852
———
289680
289704
———
00

OF THE ELLIPSIS OR OVAL.

DEF. I.

An Ellipſis is a curve-lined figure, formed by the motion of two right lines, the ſum of which is every where the ſame, and their ends fixed in two points of a given right line, equal to the ſum of the two moving lines, quite round about that given line.

Theſe two moving lines continually vary the meaſure of each, as their poſition varies, and the angle they form with one another; like a cord or meaſuring line, the two ends of which are fixed in two points at ſome diſtance one from the other, and the line ſo ſtretched out upon the ground with a pin or ſtake, which marks out a curve line, more or leſs round as theſe points are nearer or farther diſtant.

II.

The circumference, or periphery, of the ellipſis is the curve line formed or marked out by the moving end

of the two lines: as ABDEG: the moving lines are represented by FA and AH, FD and DH.

DEF. III.

The long diameter, or transverse axis, is the given right line round which the moving lines are carried, and which is equal to the sum of their measures: as BG. It is the greatest length of the ellipsis.

IV.

The centre is the middle point of the long diameter: as C.

V.

The focus is any one point of the long diameter, where one of the ends of one of the moving lines is fixed: as F or H. The two foci are equally distant from the centre.

VI.

The short diameter, or conjugate axis, is a right line perpendicular to the long diameter drawn thro' the centre, and terminated both ways by the circumference: as AE. It is the greatest breadth of the ellipsis.

VII.

The focal distance is the distance of either focus from the centre: as FC or CH.

PROB. XX.

Having the long diameter and focal distance, to describe or make an ellipsis.

RULE I.

Take a cord, or line, a little longer than the ſum of the long diameter, and focal diſtance twice taken: tie the two ends of it in a knot, ſo as the cord thus doubled may be exactly equal to one half of the aforeſaid ſum, above the knot: fix a pin in each focus; lay the cord about the two pins, cloſe upon the ſurface where the ellipſis ſhould be made, and there keeping it, ſtretch and draw it quite round with another pin in the doubling, from one end of the diameter to the other, and about again, on the other ſide, to the firſt end: as the moving triangle AFH or FDH.

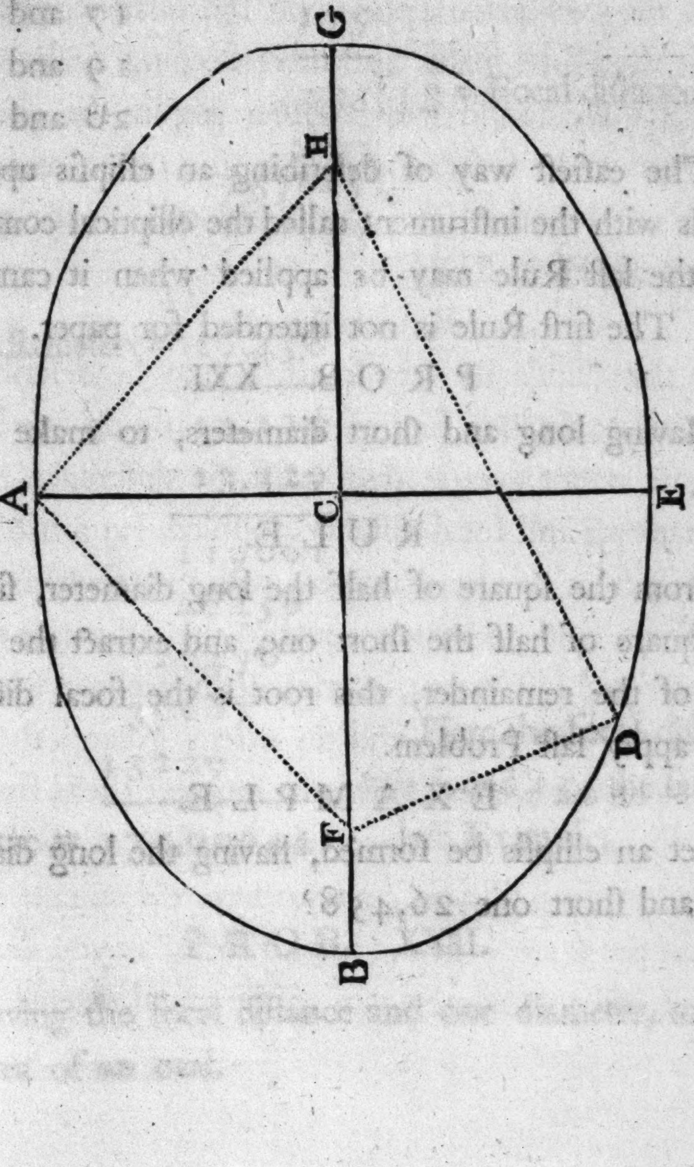
RULE II.

If the ellipſis be very ſmall, it may be deſcribed on paper, thus; fix one foot of the compaſſes in one focus; with an extent greater than half the difference of the the diameter and focal diſtance doubled, cut a ſmall arch on both ſides of the diameter; with one foot in the other focus, and an extent equal to the difference of the diameter and laſt extent, interſect theſe arches; with one foot in the firſt focus, and an extent greater than the firſt extent, cut arches as before; interſect theſe, from the other focus, with the reſt of the meaſure of the diameter; go on thus, till you come to have the extents equal; then do the ſame with the ſecond focus, as you did with the firſt, till you come again to the equal extents. Join

all theſe points of interſection with a ſteady hand; and you have done.

EXAMPLE.

Let it be required to deſcribe an ellipſis having the long diameter 3 feet 4 inches, and focal diſtance 1 foot 3 inches?



Long diameter = 40	Extents.
Focal distance doubled = 30	7 and 33
	9 and 31
Difference = 2)10	11 and 29
Half the difference = 5	13 and 27
	15 and 25
	17 and 23
	19 and 21
	20 and 20

The easiest way of describing an ellipsis upon paper is with the instrument called the elliptical compasses: but the last Rule may be applied when it cannot be had. The first Rule is not intended for paper.

PROB. XXI.

Having long and short diameters, to make an ellipsis.

RULE.

From the square of half the long diameter, subtract the square of half the short one, and extract the square root of the remainder, this root is the focal distance: then apply last Problem.

EXAMPLE.

Let an ellipsis be formed, having the long diameter 40, and short one 26,458?

2)
Long diameter = 40
20
20
Square = 400
175
225(15 = Focal distance.
25)125
125
2)
Short diameter = 26,458
13,229
13,229
119061
26458
26458
39687
13229
Square = 175,006441

Here the focal distance is found 15, the same as last Example.

PROB. XXII.

Having the focal distance and one diameter, to find the area of an oval.

RULE.

If the long diameter be given; from the ſquare of its half, ſubtract that of the focal diſtance, and extract the ſquare root of the remainder, it is half the ſhort diameter: multiply the two diameters together, and the product by ,785398. If the ſhort diameter be given; to the ſquare of its half, add the ſquare of the focal diſtance, and extract the ſquare root of the ſum, it is half the long diameter; then multiply &c. as before.

EXAMPLE I.

What is the area of an ellipſis whoſe long diameter is 517, and focal diſtance 170?

```
2)517
  ------
  258,5
  258,5
 -------
  12925
 20680
12925                 170
5170                  170
--------             -----
66822,25              119
                      17
                     -------
                     28900
                     66822,25
                     --------
                     37922,25
```

```
     37922,25(194,7363
     .  .  .
     1                    2
29)279          389,4727 = Short diameter.
   261
 384)1822
     1536
3887)28625       Short diameter = 389,4727
     27209       Long diameter  =      517
38943)141600                     27263089
      116829                     3894727
389466)2477100                 19473635
       2336796                201357,3859
        140304                   ,785398
                              16108590872
                             18122164731
                             6040721577
                           10067869295
                           16108590872
                          14095017013
               Area = 158145,6881710882
```

EXAMPLE II.

What is the area of an oval, whose short diameter is 389,4727, and focal distance 170?

Sq. of $\frac{1}{2}$ the short diam. = 37922,25
Sq. of the focal distance = 28900

Sum = 66822,25 = Sq. of $\frac{1}{2}$ the long diam.

Hence, the long diameter is 517, which multiplied &c. will give the same area as in last Example.

The area of an elliptical ring is found by subtracting the area of the inner ellipsis from that of the whole or outer ellipsis, as is done with the circular ring, p. 150.

PROB. XXIII.

With one diameter given (either the long or short one) to make an ellipsis of any proposed content, not exceeding three fourths of the square of the diameter.

RULE.

Divide the content by ,785398, and the quotient by the given diameter, the last quotient is the other diameter: then lay out the ellipsis by Prob. XXI.

EXAMPLE.

Let it be required to lay out 158145 square links in an ellipsis, the long diameter of which is 517 links?

,785398)158145,000000(201356,5
1570796
1065400
785398
2800020
2356194
4438260
3926990
5112700
4712388
400312

517)201356,5(389,4727=Short diam. as before.
1551
4625
4136
4896
4653
2435
2068
3670

An elliptical ring may be laid out around an inner ellipsis, by adding the area of the ellipsis to that of the ring proposed, and by this sum finding the other dia-

meter, as above, by the diameter given, or the sum of the measures of the diameter, and twice the breadth of the ring, which is the difference of the outer diameter found, and inner diameter given; I say the outer diameter found, because the areas are to one another, as the squares of the diameters, if you take either the two long ones, or the two short. After what has been said about finding the areas of the circular and elliptical rings, I should think any more Examples needless to explain this.

I shall conclude this Third Part, with shewing how to reduce any kind of land measures to any other, as Scots to English &c.

PROB. XXIV.

To reduce English measure to Scots, or Scots to English measure.

RULE.

Multiply the English measure by ,7869406 (or for a small piece by ,787). Divide the Scots measure by the same decimal.

EXAMPLE I.

How many acres &c. Scots in 58 acres, 3 roods 34,88 poles English?

4|0)3|4,88

4)3872

5896800 Links square.

```
 ,7869406          A. 46.40431,33
    58968                      4
 ---------         --------------
 62955248          R.  1.61725
47216436                      40
70824654           --------------
62955248           F.  24.69000
39347030
-------------
4640431,33008
```

A. R. F.
Answer 46 : 1 : 24,69

EXAMPLE II.

How many acres &c. English in 7 acres, 1 rood, 23,48 falls Scots?

```
4|0)2|3,48          ,787)739675,000(940000
  4)1587                 7083
   ------                -----
   739675                 3137

      A. 9.4
           4
         ---
      R. 1.6
          40
         ---
      P. 24.0
```

A. R. P.
Answer 9 : 1 : 24

That the proportion of the English acre to the Scots acre, and consequently any measure of the one to the correspondent measure of the other either in acres or parts of an acre, is As ,7869406 to 1. will appear by the following work.

English Inches } 37,2
in a Scots Ell } 37,2

744
2604
1116

Square Inches 1383,84 in a square Ell.
Square Ells 5760 in a Scots Acre.

830304
968688
691920

Square Inches 7970918,4 in an Acre Scots.

English Inches in } 36
an English Yard. } 36

216
108

Square Inches 1296 in a Yard square.
Square Yards 4840 in an Acre English.

5184
10368
5184

Square Inches 6272640 in an English Acre.

7970918,4)6272640,00(,7869406 = Proportional number required, for the multiplier and divisor, as above.

557964288

692997120

637673472

553236480

478255104

749813760

717382656

324311040

318836736

547430400

In the same manner may be found multipliers and divisors for reducing any other land measures; if you bring them to the terms in which both measures that are to be reduced agree, and then divide the least by the greatest.

PART IV.

DIVISION.

UNDER this title I propoſe to give only ſuch rules, for dividing any piece of ground in any ratio, or proportion required, as can be eaſily and ſurely applied in real practice, and the truth of them proven, without having any recourſe to the meaſures of a plan: but taking all the meaſures upon the ground that is to be divided; and repreſenting the figure of it by a good eye draught, or rough sketch, as directed by Prob. XXV. of Part I. which is no farther uſeful, but for this very purpoſe, to prevent miſtaking one line for another, and ſhewing the ſituation of the ſeveral parts; as in finding the contents of ſurveys by the graphometer &c.

All the dividing lines are to be ſtaked, as in laying out ground, for dividing is only laying out the ſeveral parts.

When there are off-ſets upon any of the ſides, their contents are to be diſpoſed of by the two following Problems, with which I ſhall begin.

PROB. I.

How to diſpoſe of the off-ſets upon the ſides that are not to be divided.

RULE.

If they are all without the ſide, the ſum of their contents muſt be ſubtracted from the ſhare that is to ly next them; if all within, it muſt be added: if ſome are without, and ſome within, the difference muſt be added or ſubtracted, as the ſum of the inſide or outſide off-ſets is greateſt.

PROB. II.

How to diſpoſe of the off-ſets upon the ſides that are to be divided.

RULE.

Mark out the ends of the dividing lines, in proportion to the area of the right-lined part of the field, upon the right-lined ſide or ſides. Do with the off-ſets from the beginning to the firſt marks, as directed above: if the ſhare thus found is the true firſt ſhare of the whole field, draw the dividing line through the marks to the boundary on both ſides: if it is greater, lay out the exceſs upon the dividing line, or diſtance of the marked ends, as a baſe, towards the beginning of the ſide or ſides, in a triangle, rectangle, or trapezoid; the innermoſt ſide of which produced to the boundary is the true dividing line: if it is leſs, lay out the difference as above towards the end of the ſide. Proceed, in the ſame manner, with the off-ſets betwixt the firſt true dividing line and the ſecond marks; betwixt the ſecond true dividing

line and the third marks &c. Allowance is likewiſe to be made for the off-ſets, which the true dividing line ſhall bring in to the ſhare, or take from it.

PROB. III.

To find the proportional parts anſwering unequal ratios.

RULE.

As the ſum of the ratios, to the whole meaſure, ſo is each particular ratio, to the part proportional. Or ſo is the ſum of the two firſt, three firſt &c.

Theſe Problems being only preparatory to the following, where Examples of them all muſt be given, I ſhall give none here.

PROB. IV.

To divide a triangle in any propoſed ratio, by right lines from one angle to the oppoſite ſide.

RULE.

Divide the ſide in the ratio propoſed, join the angular point and points of diviſion.

EXAMPLE I.

Let it be required to divide the triangle ABC, containing 4 acres, 1 rood, 23,12 poles, into 3 equal parts, by right lines proceeding from the angle A to the ſide BC?

		300	24
		500	28
		700	22
		900	16
	00	1000	00
	18	1200	
	14	1400	
	00	50	
AB = 1450			
	00	400	00
	32	600	
	20	800	
	00	1000	
		1680	
BC = 1680			

Right Hand Off-sets.	Left Hand Off-sets.	
3600	1800	
5200	3200	
5000	350	
3800		5350 on AB.
800	3200	
	5200	
18400	2000	
15750		10400 on BC.
	15750	

2650 = Difference.

Off-sets on AB	without	18400
	within	5350
	Difference =	13050

Right-lined area = 436800. Share = 145600
Diff. of Off-sets = 2650

Whole area = 439450. Share = 146483

Share of the whole	=	146483
Off-sets in the first share	=	13050
First share of right-lined part	=	133433
Second share of right-lined part	=	146483
Sum of first 2 shares	=	279916
Right-lined area	=	436800
Third share of right-lined part	=	156884

R. l. area.	BC.	Shares.	Parts of BC.
436800 :	1680 ::	133433 :	513 = 1st Part = B*b*
		146483 :	563 = 2d = C*e*
		156884 :	604 = 3d = *eb*
		436800	1680

```
            133433
               1680
           --------
           1067464
           800598
           133433
          ---------
4368|00)2241674|40(513
           21840
          -------
            5767
            4368
          -------
            1399

            146483
              1680
          ---------
           1171864
           878898
           146483
          ---------
4368|00)2460914|40(563
           21840
          -------
           27691
           26208
          -------
            1483
```

Aa = 625

ac = 170

Alt. of *abc* = 20

$$\begin{array}{r} 170 \\ 10 \\ \hline 1700 \end{array} = \Delta\, abc = \text{Deficiency} —$$

— of the 1st share, made up by Δ A*da*.

$$625)1700(2{,}7 = \tfrac{1}{2}\ \text{Alt. of A}da \qquad \begin{array}{r} 2{,}7 \\ 2 \\ \hline 5{,}5 \end{array}$$
$$\begin{array}{r} 1250 \\ \hline 450 \end{array} \qquad \text{Alt. of A}da =$$

Dividing line for right-lined 1st share is A*b*.
Dividing line for 1st ~~true~~ share is A*d*, for 2d A*e*.

Here you may see all the first three Problems exemplified. The difference of the off-sets upon AB is subtracted from the share, because the out-side sum is greatest, and consequently the first share of the triangle exceeds one third of the right-lined part: and because the first share falls short by the off-set triangle *abc*, the triangle A*da* equal to it ~~is laid out upon~~ the base A*a*. The rest of the work seems to need no explanation.

See Plate III. for the rough draught of this field.

The proof of the rule appears by the 1. of EUCLID's VI. where it is demonstrated, that triangles having the same altitude are to one another as their bases.

EXAMPLE II.

Let it be required to divide the triangle ABC, containing 3 acres, 0 roods, 24,528 poles, in the ratio

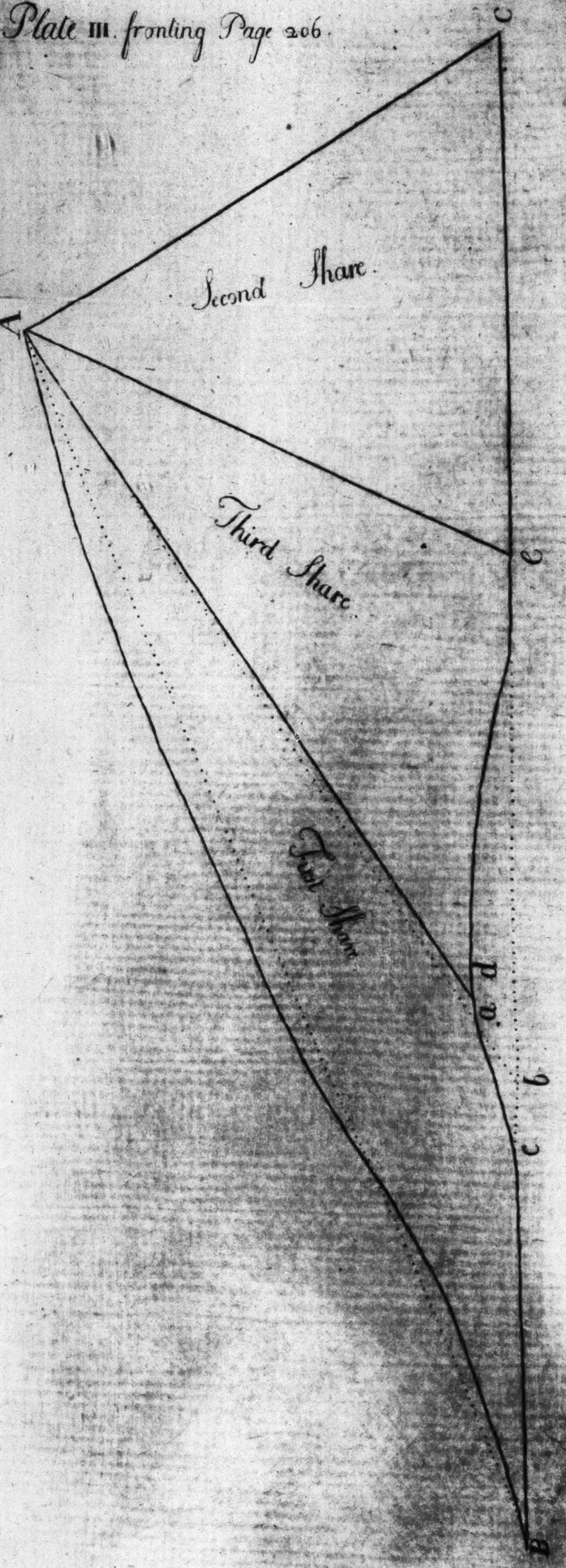
la.

m-
ub-
eft,
eds
irft
gle
The

eld.
D's
the

con-
ratio

Plate III. fronting Page 206.



of 5 . 7 . 8 . 10, by right lines from the angle B to AC? See Plate IV. Figure I.

			Off-sets.
00	00	40	
	100	52	4600
	200	48	5000
	300	60	5400
	50	54	2850
	50	46	2500
	500	38	4200
	20	32	700
	80	40	2880
	700	28	3400
	800	24	2600
	900	20	2200
	60	00	600
	960=CA.		36930 = Sum.

Right-lined area = 278400
Sum of off-sets = 36930
Whole area = 315330

Ratios.

5
7
8
10

Sum = 30 : 278400 : :

Right-lined Shares.

5 : 46400
7 : 64960
8 : 74240
10 : 92800

$$\begin{array}{r} 278400 \\ 7 \\ \hline 3|0)\,19488 0|0 \\ \hline 64960 \end{array} \qquad \begin{array}{r} 278400 \\ 8 \\ \hline 3|0)\,222720|0 \\ \hline 74240 \end{array}$$

The second term is divided by 6 to find the first share, because 5 is the $\frac{1}{6}$ of 30: and by 3 to find the fourth share, because 10 is $\frac{1}{3}$ of 30.

$$30 : 315330 :: \left.\begin{array}{r} 5 : 52555 \\ 7 : 73577 \\ 8 : 84088 \\ 10 : 105110 \end{array}\right\}\text{Whole Shares.}$$

$$\begin{array}{r} 315330 \\ 7 \\ \hline 3|0)\,220731|0 \\ \hline 73577 \end{array} \qquad \begin{array}{r} 315330 \\ 8 \\ \hline 3|0)\,252264|0 \\ \hline 84088 \end{array}$$

$$\begin{array}{c} \text{AC} \\ 30 : 960 :: \end{array} \left.\begin{array}{r} 5 : 160 \\ 7 : 224 \\ 8 : 256 \\ 10 : 320 \end{array}\right\}\text{Parts of AC.}$$

$$\begin{array}{r} 960 \\ 7 \\ \hline 3|0)\,672|0 \\ \hline 224 \end{array} \qquad \begin{array}{r} 960 \\ 8 \\ \hline 3|0)\,768|0 \\ \hline 256 \end{array}$$

52555 = Whole Share.
46400 = R. l. Share.

Differences to be made up by the Off-sets.

6155 = Difference.
8617
9848
12310

Sum = 36930 = Off-set Sum.

73577 =	Whole Share	= 84088	105110
64960 =	Right-lined Share	= 74240	92800
8617 =	Difference	= 9848	12310

First Off-set = 4600
adef = 3500
8100
First Difference = 6155
Excess of 1[st] share = 1945 = *ag*B

2)
B*a* = 646
323)1945(6
Alt. of *ag*B = 6

ad, or *ef* = 70
ae, or *df* = 50
aefd = 3500
2[d] Off. = 5000
aeih = 1500

kl = 50
ml = 40
blmk = 2000

$aeih = 1500$
3^d and 4^{th} Off. $= 8250$
$blmk = 2000$

11750
Second Difference $= 8617$

Excess of 2^d Share $= 3133$
Add *euga* to it $= 1745$

2) Whole Excess $= 4878$
$bB = 640$

$320)4878(15 = $ Alt. of bnB = Whole Excess of 2^d Share.
32

167 The 5^{th} Off-set is included in *qpkl*.

$bm = 50$	$tr = 50$
$mp = 20$	Alt. $= 30$
$mbpq = 1000$	$ctrs = 1500$

2)
$cB = 730$
$365)5310(15$ nearly.
365
1660
15 = Alt. of cvb = Deficiency of 4^{th} Share.

$mbpq = 1000$

6^{th}, 7^{th} and 8^{th} Off-ſet $= 7780$

$ctrs = 1500$

10280

Third Difference $= 9848$

Exceſs $= 432$

To which add the above 4878

The whole Exceſs of 3^{d} Share $= 5310$ =Deficiency of the 4^{th} Share.

Dividing Lines

For right-lined part *a*B, and *q*B, and *c*B.

For the whole, *g*B, and *b*B, and *v*B.

PROB. V.

To divide a triangle in any ratio, by right lines parallel to one ſide, and cutting the others.

RULE.

As the whole content, to the ſquare of the ſide to be divided, ſo is the firſt ſhare, to the ſquare of the firſt part of that ſide: and ſo is the ſum of the firſt two ſhares, to the ſquare of the ſum of firſt two parts: and ſo is the ſum of the firſt three ſhares, to the ſquare of the ſum of firſt three parts &c. Extract the ſeveral roots, ſubtract &c. and lay off the meaſures upon the ſide. Do

the ſame with the other ſide, and join the ends of theſe meaſures. Thus the right-lined part will be divided: then do with the Off-ſets as before.

EXAMPLE.

Let the triangle ABC, containing 9 acres, 1 rood, 0,832 poles, be divided into 4 equal parts by right lines parallel to BC, and cutting AB and AC?

See Plate IV. Fig. II.

00	100	00		Off-ſets.
	300	36		L. H.
	500	42		4000
	700	50		7400
	900	56		6200
	1100	48		3600
	1300	30		1440
			1300 = BA.	2320
00	400	00		1650
40	600			4200
34	800			1900
28	1000			
44	1100			Sum = 32710
52	30			
64	40			R. H.
46	30			3600
38	1300			7800
00	1400			9200
			1400 = AC.	10600
				10400
				7800
				Sum = 49400

49400
32710
Diff. = 16790

4)
Right-lined area = 911170(Share = 227792
Diff. of Off-ſets = 16790

Whole area = 927960(Share = 231990

Diff. = 4198

First share of right-lined part = 227792
2
Sum of first 2 shares = 455584
227792
Sum of first 3 shares = 683376

AB = 1300
1300
39
13
ABq = 1690000

R. l. area.	ABq.		Shares.	Parts squared.	
911170 :	1690000 : :		227792 :	422500	
			455584 :	845000	the first doubled.
			683376 :	1267500	the first tripled.

227792
1690000
2050128
1366752
227792
91117|0)384968480000|0(422500(650 = First part of AB to be laid off from A.
364468
205004
182234
227708
182234
45474

36
125)625
625

845000(919 = Sum of first 2 parts of AB.
81
181)350
181
1829)16900

650
269

1267500(1126 = Sum of first 3.
21)26
21
222)575
444
224)13100

919
207

Parts of AB.
650 = A*d*.
269 = *de*.
207 = *ef*.
174 = *f*B.

AC = 1400
1400

56
14

ACq = 1960000

ACq. 227792 : 490000
911170 : 1960000 : : 455584 : 980000
683376 : 1470000

227792
1960000

1366752
2050128
227792

91117|0)44647232000|0(490000

364468

820043

490000(700 = *Aa.*

980000(990 = *Ab.*
81

189)1700
1701

1470000(1212,5
990

22)47
44

222,5

1400
1212,5

187,5

241)300
241

5900

Parts of AC,	
700	= *Aa.*
290	= *ab.*
222,5	= *bc.*
187,5	= *c*C.

Inſtead of working as above for diviſions of AC as well as for thoſe of AB, parallels might have been drawn to BC thro' the points *d*, *e* and *f*. But this method will be found not ſo exact in the field,

For the proof of the Rule, see EUCLID VI. 2. and 19. where it is demonstrated, that right lines cutting two sides of a triangle and parallel to the third, cut the sides proportionally: and that similar triangles are to one another as the squares of their homologous sides. That is Δ ABC : ABq : : Δ Ada : Adq, and Δ ABC : ABq : : Δ Aeb : Aeq. But Δ Ada is $\frac{1}{4}$ of Δ ABC and Δ Aeb $\frac{1}{2}$ of Δ ABC, consequently Δ Aeb is likewise double of *adeb*, which therefore is also $\frac{1}{4}$ of Δ ABC &c. Let us now come to the Off-sets, and make all ready for drawing the true dividing lines.

$$\begin{array}{r} ak = 125 \\ lm = \underline{\ \ 89} \\ 2)\underline{214} \\ 107 \end{array} \qquad \begin{array}{r} \text{Alt. of } aklm = 40 \\ \underline{107} \\ aklm = 4280 \end{array} \qquad \begin{array}{c} \text{Diff. of shares} \\ 4198 \end{array}$$

$$\begin{array}{rr} aklm = & 4280 \\ \text{First Off-set of AC} = & 4000 \\ \text{Difference of shares} = & \underline{4198} \\ & 12478 \end{array}$$

The three last Off-sets of BA = 28800

$$\begin{array}{r} gd = 50 \\ hi = \underline{70} \\ 2)\underline{120} \\ 60 \end{array} \qquad \begin{array}{r} \text{Alt. of } ghid = 50 \\ \underline{60} \\ ghid = 3000 \end{array} \qquad \begin{array}{r} \underline{3000} \\ \left.\begin{array}{r} 31800 \\ \underline{12478} \end{array}\right\} \begin{array}{l} = \text{Off. without.} \\ \text{——within.} \end{array} \\ 19322 = \text{Excess of} \\ \text{A}im = nim. \end{array}$$

$$\begin{array}{l} \quad 2) \\ im = \underline{815} \\ \quad 407,5)19322,0(47,5 = \text{Alt. of } nim. \\ \qquad\qquad \underline{16300} \\ \qquad\qquad 30220 \end{array}$$

True dividing line for first share is *nm*.

Off-sets of *adeb* on both sides nearly equal.

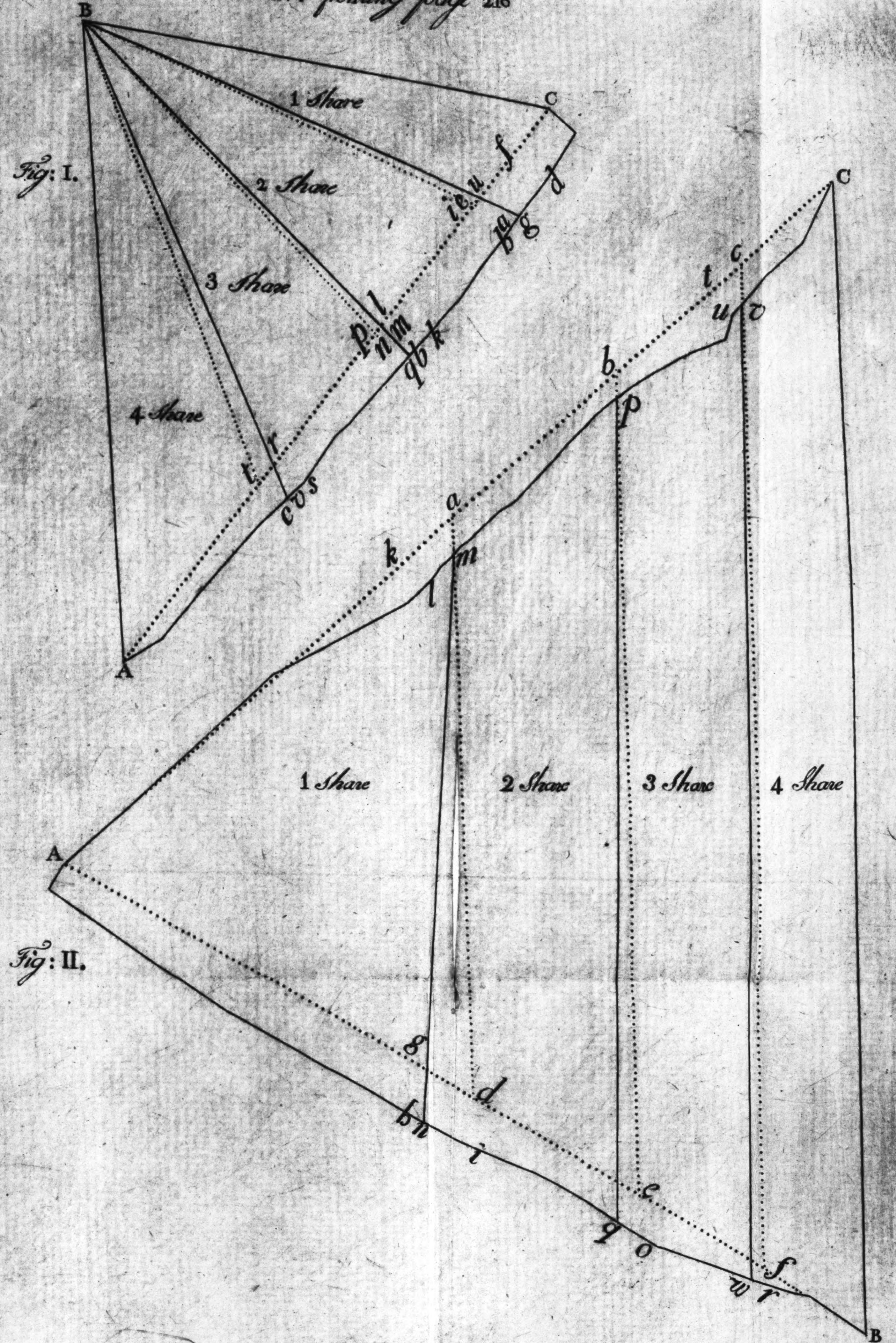
$adeb = 227792$	$anob = 247114$
$nim = \underline{\ \ 19322}$	$\text{Share} = \underline{231990}$
$anob = 247114$	$\text{Excess} = \ 15124$ of $mnop = pqo$.

$$\begin{array}{l} \quad 2) \\ po = \underline{1152,3} \\ \quad 576,15)15124,00(26,2 = \text{Alt. of } opq. \\ \qquad\qquad \underline{115230} \\ \qquad\qquad\ \ 360100 \\ \qquad\qquad\ \ \underline{345690} \\ \qquad\qquad\ \ \ 14410 \end{array}$$

The true dividing line for second share is *pq*.

$eo = 46$	Alt. of $eorf = 227$
$rf = \underline{18}$	$\underline{32}$
$2)\overline{64}$	454
$\overline{\ 32}$	$\underline{681}$
	$eorf = 7264$

Plate IV. fronting page 216



uv = 20 *ct* = 68 2)88 44	Alt. = *tu* = 46 44 —184 184 *ctuv* = 2024	4th, 5th, 6th and 7th Off. of AC = 9010 *ctuv* = 2024 Diff. of Shares = 4198 15232

opq = 15124
eorf = 7264

without 22388
within 15232

7156 = Exceſs of *pqrv* = Deficiency — — of *vr* BC = *vwr*.

2)
rv = 1374

687)7156(10,4 = Alt. of *vwr*.
687
286

The true dividing line for third ſhare is *vw*.

In the following Problems, I ſhall take no more examples of off-ſets, but ſuppoſe the way of doing with them well enough explained by the three laſt examples.

PROB. VI.

To divide any parallelogram, viz. ſquare, rectangle, rhombus, or rhomboides, in any ratio propoſed by right lines parallel to two of the ſides, and cutting the others.

RULE.

Divide these other sides in the proposed ratio, and join the points of division.

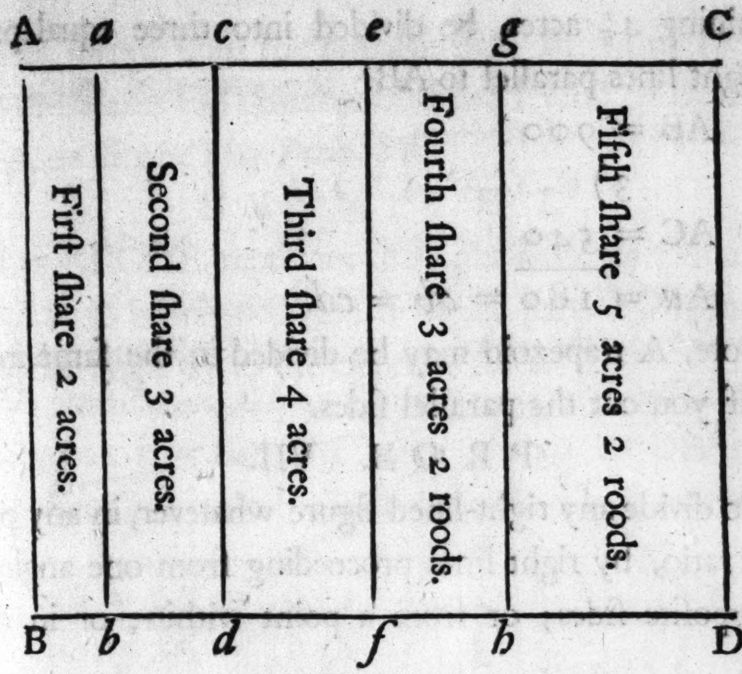
EXAMPLE I.

Let the rectangular park ABCD, containing 18 acres, be divided, so as the first share may be 2 acres, the second 3, the third 4, the fourth $3\frac{1}{2}$, and the fifth $5\frac{1}{2}$ acres, by right lines parallel to AB, and cutting AC?

Let AC be 1500 links, and AB = 1200, and the content is 1800000 square links, as above.

$$18 : 1500 :: \begin{cases} 2 : 166\frac{2}{3} = Aa = Bb. \\ 5 : 416\frac{2}{3} = Ac = Bd. \\ 9 : 750 = Ae = Bf. \\ 12{,}5 : 1041\frac{2}{3} = Ag = Bh. \end{cases}$$

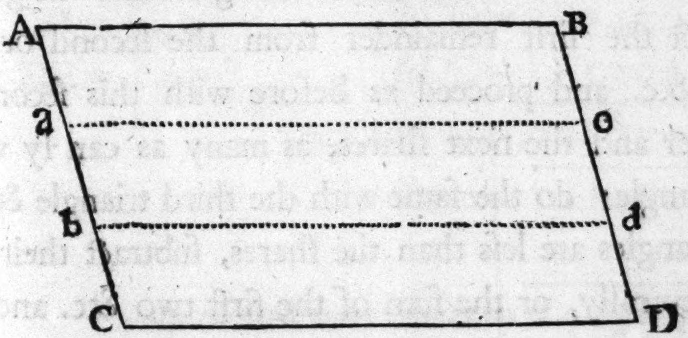
$$\begin{array}{r} 1500 \\ \underline{5} \\ 18)\overline{7500} \\ \overline{416{,}666} \end{array} \qquad \begin{array}{r} 12{,}5 \\ \underline{1500} \\ 18)\overline{18750{,}0} \\ \overline{1041{,}666} \end{array}$$



Here the ſum of firſt and ſecond ratios is uſed inſtead of the ſecond, that of the firſt three inſtead of the third &c. that the meaſures may be laid off from the beginning of the line; which is the ſureſt way with the chain: and with a ſcale too.

The proof of this Problem is by EUCLID VI. and 1.

EXAMPLE II.



Let the incloſure ABCD of a diamond-like figure,

containing $4\frac{1}{2}$ acres, be divided into three equal parts, by right lines parallel to AB?

$$AB = 900$$
$$3)$$
$$AC = \underline{540}$$
$$Aa = 180 = ab = cd.$$

Note, A trapezoid may be divided in the ſame manner, if you cut the parallel ſides.

PROB. VII.

To divide any right-lined figure whatever, in any propoſed ratio, by right lines proceeding from one angle to the oppoſite ſides; or from a point within; or in one ſide.

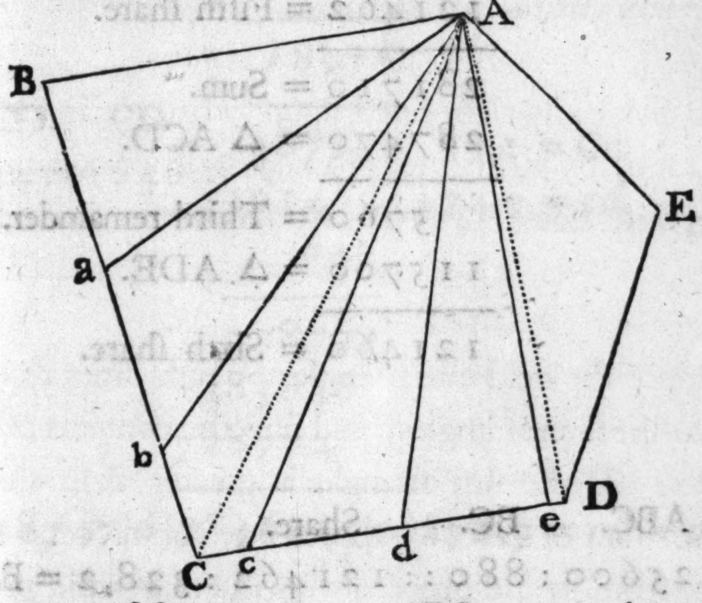
RULE.

Reduce the field into triangles by right lines from the point to the ſeveral angles; and find the contents of theſe triangles. If the firſt triangle is greater than the firſt ſhare, or ſum of firſt two ſhares &c. ſubtract the ſhare, or ſum, from it, and proceed with the ſhares and remainder by Prob. IV. If the ſecond triangle is likewiſe greater, ſubtract the firſt remainder from the ſecond or third ſhare &c. and proceed as before with this ſecond remainder and the next ſhares, as many as can ly within the triangle: do the ſame with the third triangle &c. If the triangles are leſs than the ſhares, ſubtract their contents ſeverally, or the ſum of the firſt two &c. and pro-

ceed as above with the remainders. In ſhort, proceed with the ſhares and differences of the ſeveral triangles and ſhares, as directed by Prob. IV.

EXAMPLE I.

Let ABCDE repreſent the right-lined part of a field which is to be divided into 6 equal parts by right lines from the angle A to the oppoſite ſides.



BC = 880
CD = 646
DE = 547

ABC = 325600
ACD = 287470
ADE = 115700

ABCDE = 6)728770

Share = 121462
2

Sum of firſt two ſhares = 242924

$$
\begin{array}{rl}
\Delta\,ABC = & 325600 \\
& \underline{242924} \text{ Sum of first 2 shares.} \\
& 82676 = \text{First remainder.} \\
& \underline{121462} = \text{Third share.} \\
& 38786 = \text{Second remainder.} \\
& 121462 = \text{Fourth share.} \\
& \underline{121462} = \text{Fifth share.} \\
& 281710 = \text{Sum.} \\
& \underline{287470} = \Delta\,ACD. \\
& 5760 = \text{Third remainder.} \\
& \underline{115700} = \Delta\,ADE. \\
& 121460 = \text{Sixth share.}
\end{array}
$$

$$
\begin{array}{lll}
\text{ABC.} & \text{BC.} & \text{Share.} \\
325600 : 880 :: 121462 : 328{,}2 = Ba.
\end{array}
$$

$$
\begin{array}{r}
880 \\
\hline
971696 \\
971696 \\
\hline
106886560
\end{array}
$$

3256|00)1068865|60(328,2 = B*a* = *ab*.

9768

9206
6512

26945
26048

8976

ACD.	CD.		
		38786 :	87,1 = C*c*.
287470 :	646 ::	121462 :	272,9 = *cd*.

646

728772
485848
728772

28747|0)7846445|2(272,9 = *cd* = *de*.

57494

209704
201229

84755
57494

272612

38786
646

232716
155144
232716

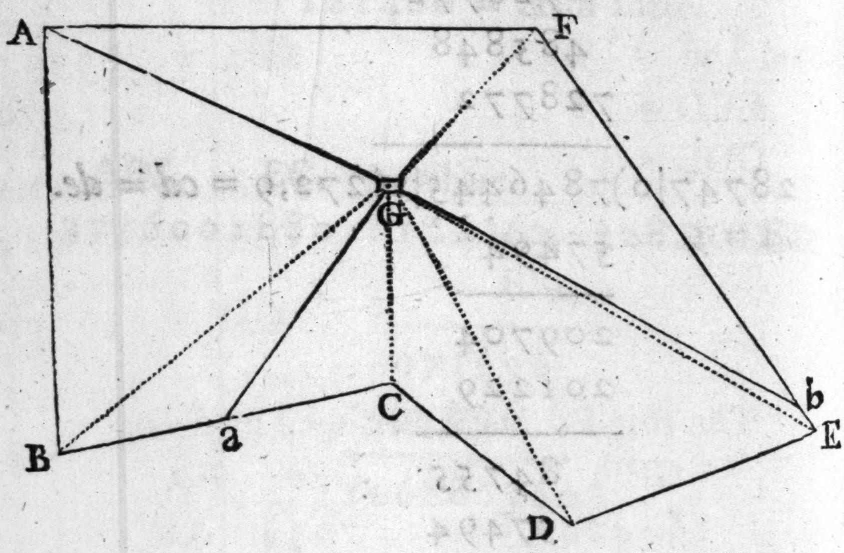
28747|0)2505575|6(87,1 = Cc.
229976

205815
201229

4586

Thus the ſeveral ſhares are AB*a*, A*ab*, A*b*C*c*, A*cd*, A*de* and A*e*DE, each $\frac{1}{6}$ of the whole field ABCDE.

EXAMPLE II.



Let the field ABCDEF be divided into 3 equal parts, by right lines, from the middle of a pond within it, as G.

AB = 760	GAB = 226480
BC = 586	GBC = 101964
CD = 400	GCD = 65450
DE = 464	GDE = 156860
EF = 884	GEF = 164424
FA = 862	GAF = 122404

3)837582

Share = 279194

Share = 279194
GAB = 226480
First Diff. = 52714

GBC = 101964
G*B**a* = 52714
Second Diff. = 49250

Second Diff. = 49250
GCD = 65450
GDE = 156860
G*a*CDE = 271560

Share = 279194
271560
Third Diff. = 7634
GEF = 164424
G*b*F = 156790

The part E*b* might be found by saying

GEF. EF. GE*b*. E*b*.
164424 : 884 : : 7634 : 41

GEF.	EF.	GF*b*.	F*b*.
164424 :	884 : :	156790 :	843

$$\begin{array}{r} 156790 \\ 884 \\ \hline 627160 \\ 1254320 \\ 1254320 \\ \hline 164424)138602360(843 = Fb. \\ 1315392 \\ \hline 706316 \\ 657696 \\ \hline 486200 \end{array}$$

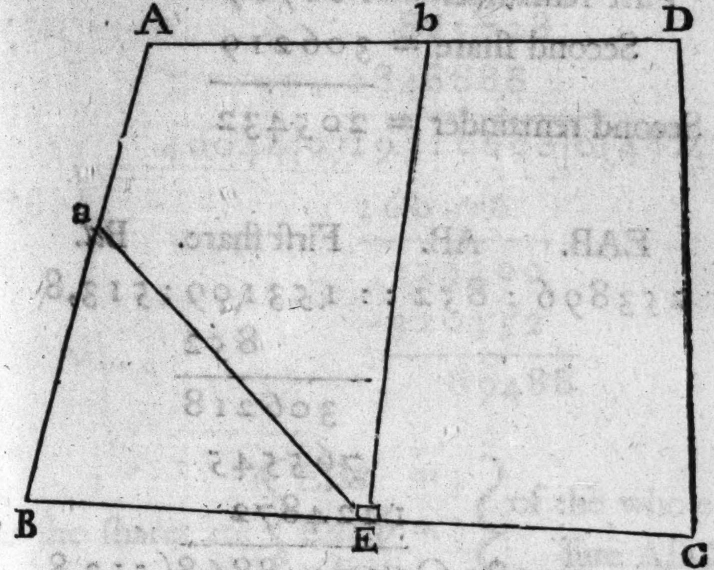
GBC.	BC.	GB*a*.	B*a*.
101964 :	586 : :	52714 :	303

$$\begin{array}{r} 52714 \\ 586 \\ \hline 316284 \\ 421712 \\ 263570 \\ \hline 101964)30890404(303 = Ba. \\ 305892 \\ \hline 301204 \end{array}$$

Thus the ſeveral ſhares are GAB*a*, G*a*CDE*b*, an GbFA, each $\frac{1}{3}$ of ABCDEF.

EXAMPLE III.

Let the inclosure ABCD be divided in the ratio of 1, 2, 3, by right lines, from the gate E, in the side BC.



		Triangles.
BA = 852		EAB = 253896
AD = 940		EAD = 400440
CD = 896		ECD = 264320
		Area = 918656

Shares.

$$6 : 918656 :: \begin{array}{l} 1 : 153109 \\ 2 : 306219 \\ 3 : 459328 \\ \hline \quad\; 918656 \end{array}$$

EAB = 253896
First ſhare = 153109

First remainder = 100787
Second ſhare = 306219

Second remainder = 205432

EAB.	AB.	First ſhare.	B*a*.
253896 :	852 : :	153109 :	513,8

```
                 153109
                    852
                 ------
                 306218
                765545
              1224872
              ---------
253896)130448868(513,8
        1269480
        -------
         350086
         253896
         ------
         961908
         761688
         ------
         200220
```

EAD. AD. 2[d] Rem. A*b*.

```
400440 : 940 : : 205432 : 482,2
                            940
                        -------
                         821728
                       1848888
                      ---------
  40044|0)19310608|0(482,2
             160176
           --------
            329300
            320352
           --------
              89488
```

Thus the ſhares are $\left\{\begin{array}{l} EBa = \frac{1}{6} \\ EaAb = \frac{1}{3} \\ EbDC = \frac{1}{2} \end{array}\right\}$ of the whole incloſure ABCD.

PROB. VIII.

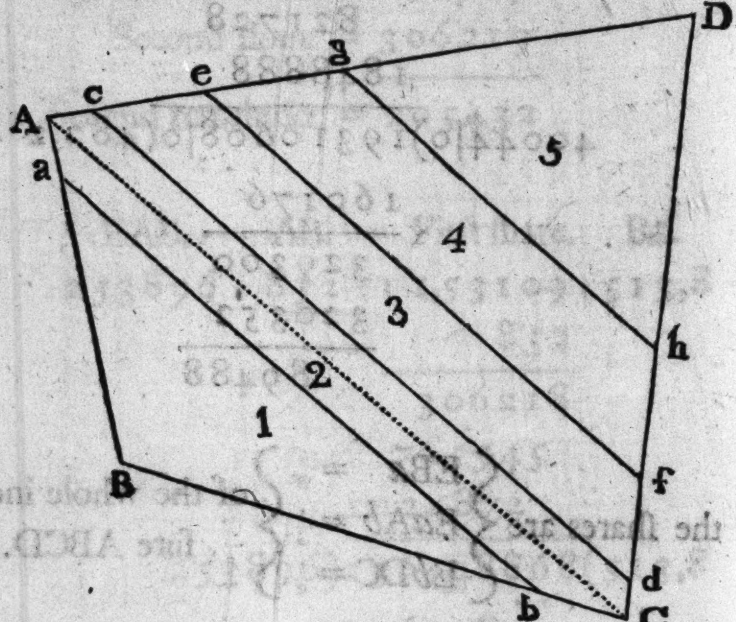
To divide a quadrangle in any propoſed ratio by right lines parallel to the diagonal.

RULE.

Divide one of the triangles by Prob, V. for as many ſhares, beginning with the firſt, as will ly within it; and the other triangle in the ſame manner, beginning with the laſt: leaving that ſhare which is not taken in, at the diagonal.

EXAMPLE.

Let the trapezium ABCD be divided into 5 equal parts by right lines parallel to AC.



BA = 316	ABC = 65170
BC = 473	ADC = 152980
DA = 584	5)218150
DC = 545	Share = 43630

AB = 316
316
1896
316
948
ABq = 99856

ABC. ABq. First Share.
65170 : 99856 :: 43630 : 66851,5

```
          43630
        -------
        299568
       599136
      299568
     399424
     -----------
6517|0)435671728|0(66851,5(258,6 = Ba.
     39102         4
     -----       ----
     44651     45)268
     39102        225
     -----       -----
      55497  508)4351
      52136      4064
      -----      ----
       33612      287
       32585
       -----
       10278
```

```
    BC = 473
         473
        ----
        1419
       3311
      1892
     ------
BCq = 223729
```

ABC. BC*q*. Share.

65170 : 223729 :: 43630 : 149782

43630

671187
1342374
671187
894916

6517|0)976129627|0(149782(387 = B*b*.

6517 9

32442 68)597
26068 544

63749 767)5382
58653 5369

50966 13
45619

53472
52136

13367

AD = 584
584
2336
4672
2920
ADq = 341056

ADC. ADq.
152980 : 341056 : : 43630 : 97269,4
43630
1023168
2046336
1023168
1364224
15298|0)1488027328|0(97269,4(311,9=Dg.
137682 61(72
111207 61
107086 621)1169
41213 621
30596 54840
106172
91788
143848
137682
6166

```
   97269,4
         2
  ---------
  194538,8(441,6 = Dc.
  16
  ---
84)345
   336
  ----
881)938
    881
   ----
    5780

         97269,4
               3
        --------
        291808,2(540,2 = Dc.
        25
        ---
    104)418
        416
       ----
   1080)20820

       DC = 545
            545
           ----
           2725
          2180
         2725
        ------
 DCq = 297025
```

ADC. DC*q*.

152980 : 297025 : : 43630 : 84711,6

```
            43630
          -------
           891075
         1782150
         891075
       1188100
       ---------------
15298|0)1295920075|0(84711,6
         122384         108880
         ------         107086
          72080         ------
          61192          17947
         ------          15298
         108880          -----
                          26495
```

$84711{,}6(291{,}5 = Dh.$

```
       84711,6(291,5 = Dh.
       4
       --
   49)447
      441
      ----
  581)611
      581
      ----
      3060
```

```
      84711,6
           2
   ___________
  169423,2(411,6 = Df.
   __ __
  81)94                 84711,6
     81                       3
   _____              _________
  821)1323            254134,8(504,1 = Dd.
      821            _________
      _____           1004)4134
      50220                4016
                          ______
                           11880
```

PROB. IX.

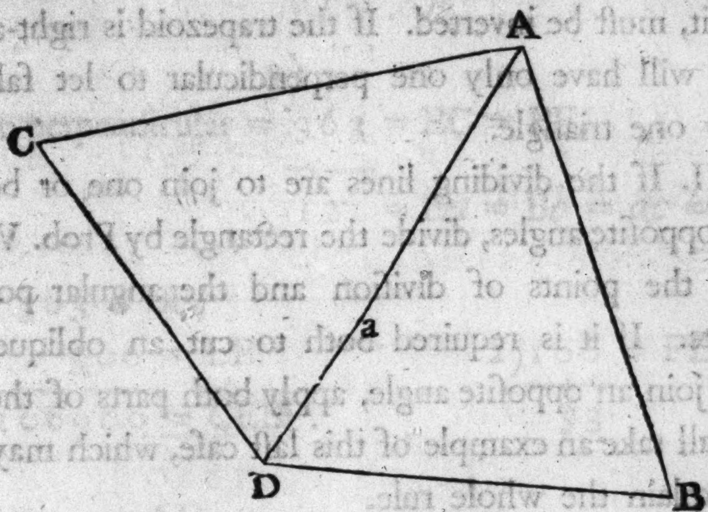
To divide a quadrangle in any ratio, by right lines joining two oppoſite angles.

RULE.

Divide the diagonal in the propoſed ratio, and join the angular points and points of diviſion.

EXAMPLE.

Let the field, represented by ABCD, be divided into two halves, by right lines joining the angles A and D.



$$\frac{2)\ CB = 652}{Ca = 326 = aB.}$$

PROB. X.

To divide a trapezoid in any ratio, by right lines cutting the oblique ſides; or joining one or two oppoſite angles.

RULE.

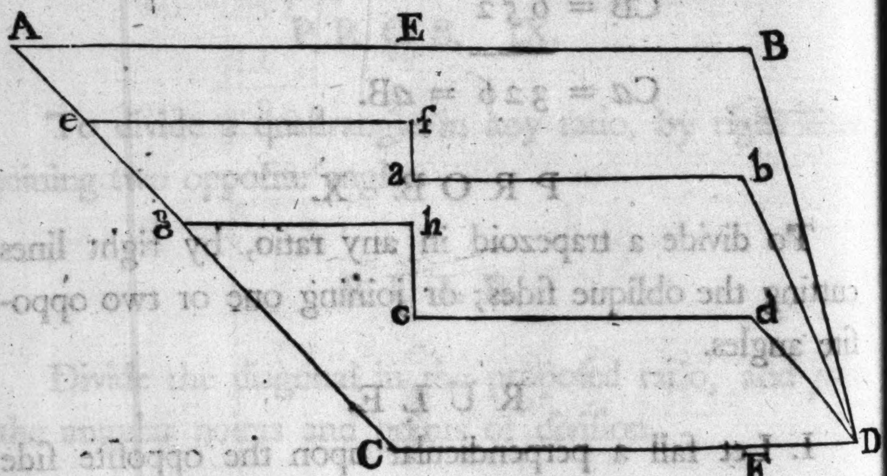
I. Let fall a perpendicular upon the oppoſite ſide from each of the obtuſe angles, if it is not right-angled: theſe will reſolve it into a rectangle and two triangles:

divide them by Prob. VI. and V. and join the points of division on the perpendiculars: but if one triangle ly contrary to the other, the order of the ratios, in dividing it, must be inverted. If the trapezoid is right-angled, you will have only one perpendicular to let fall, and only one triangle.

II. If the dividing lines are to join one or both of the opposite angles, divide the rectangle by Prob. VI. and join the points of division and the angular point or points. If it is required both to cut an oblique side, and join an opposite angle, apply both parts of the rule. I shall take an example of this last case, which may serve to explain the whole rule.

EXAMPLE.

Let the field, represented by the trapezoid ABCD,



be divided into 3 equal parts, by right lines cutting the side AC, and joining the angle D.

EB = 300
AE = 344
FD = 106

3)

Alt. or perpendicular = 363 = EC = BF.

121 = E*a* = B*b* = *ac* = *bd*.

363 = BF.
300 = EB.

108900 = CEBF.

2)106 = FD.
53

2)344 = AE.
172
363 = EC
516
1032
516
62436 = ACE.

363 = BF.
53
1089
1815
19239 = BFD.

CEBF = 108900
ACE = 62436
BFD = 19239

190575 = ABCD.

EC = 363
363
1089
2178
1089
EC*q* = 131769

AE = 344
344
1376
1376
1032
AE*q* = 118336

EC*q* = 131769
AE*q* = 118336
AC*q* = 250105

ACE. EC*q*.
62436 : 131769 :: 20812 : 43923

20812
263538
131769
1054152
263538

62436)2742376428(43923(209,6 = C*h*.
249744
244936
187308
576284
561924
143602
124872
187308

409)3923
3681
418)24200

ACE. ACq.

3)62436 : 250105 : : 20812 : 83368,3

```
          20812
         ------
         500210
        250105
       2000840
      500210
     ----------
62436)520518526o(83368,3(288,7 = Cg.
      499488        4
      ------       ---
       210305   48)433
       187308      384
       ------     -----
        229972 568)4968
        187308     4544
        ------     -----
        426646      42430
        374616
        ------
        520300
```

```
AB = 644                 525
CD = 406                 363
     ----               -----
   2)1050                1575
     ----               3150
      525              1575
                       ------
           ABCD = 190575
```

See p. 239.

```
83368,3
      2
166736,6(408,3 = Ce.
808)6736
    6464
     27260
```

```
43923
    2
87846(296,4 = Cf.
4
49)478
   441
586)3746
    3516
     230
```

Thus the several shares are the polygons C*ghcd*D, *gefab*D*dch*, and *e*ABD*baf*, each of them one third part of the trapezoid ABDC.

It is obvious, that the leading problems here given, for dividing ground, are the IV. V. and VI. the VII. and IX. are applications of the IV. as the VIII. is of the V. and the X. of all three together. It would be easy to add more Problems for dividing particular figures; but I shall rather chuse to add one, for dividing a field of any common figure whatever, which can always be applied, and is in most cases the easiest, surest and best method in practice.

PROB. XI.

To divide a field of any irregular figure, in any ratio proposed, without being confined to any point or line of it.

RULE.

Meaſure a part nearly equal, in your judgment, to the firſt ſhare; then add to it, or cut off from it, as much as will make the ſhare exact: do the ſame for the ſecond ſhare, for the third &c. much the ſame way as with the off-ſets &c. in the examples of the IV. and V. Problems of this Part, and as directed by Prob. II. Examples are needleſs.

Before we proceed to the diviſion of the circle and regular polygon, it will be neceſſary to premiſe the following Problem.

PROB. XII.

To find the centre of a circle upon the ground.

RULE.

Find the length of the radius by Prob. XIV. of Part III. at any point of the circumference, ſet up a pole; draw a chord from it, and ſet another pole at the end of this chord; at this pole raiſe a perpendicular to the chord reaching to the circumference, which will be another chord: from the end of this perpendicular, meaſure a right line towards the firſt pole equal to the radius, the end of this meaſure is the centre. For proof, try if the ſame meaſure reaches to both the poles: this will ſhow the truth of the work.

For the rule, ſee EUCLID III. 31. demonſtrating a ſegment containing a right angle, a ſemicircle.

PROB. XIII.

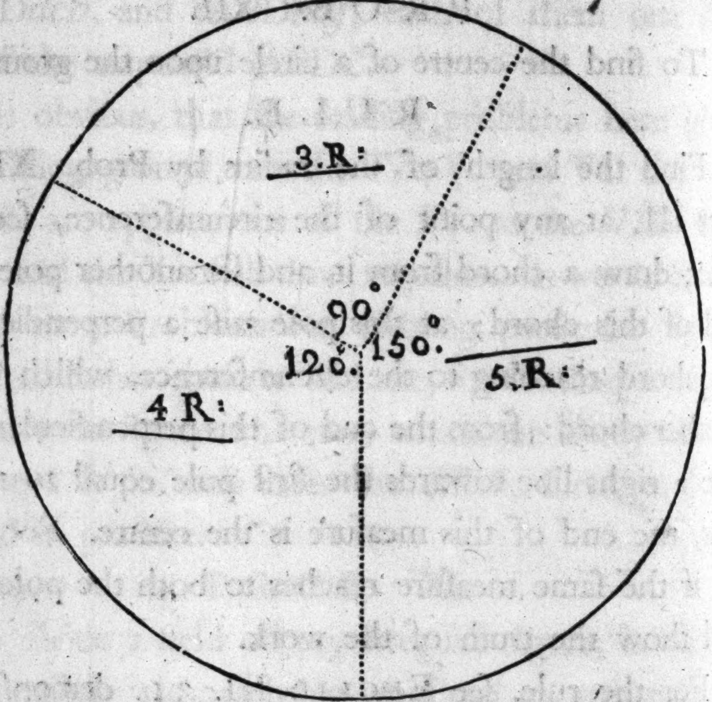
To divide a circle in any ratio, by ſectors.

RULE.

Divide the meaſure of the circumference in the propoſed ratio: then, as the whole circumference, to $360°00'$, ſo is each of the ſeveral parts, to the meaſure of the angle of each ſector required, in degrees and decimals, which may be reduced to minutes. Make theſe angles at the centre, and draw the ſeveral radii.

EXAMPLE.

Let a circle, containing 3 acres, be divided into ſec-



tors, one of 3 roods, one of 1 acre, and the third of 5 roods.

```
                                 2)
,7854)300000,0000(381971(618 = Diameter.
       23562           36       309 = Radius.
       -----          -----
        64380     121)219
        62832          121
        -----          ----
         15480          9871
          7854
         -----
          76260
          70686
          -----
           55740
           54978
           -----
             7620

          3,1416
             618
          -------
          251328
          31416
         188496
         --------
 12 : 1941,5088 :: 3:485,3772 } Parts of the
 Circumference.     4:647,1696 }    Circle.
```

1941,5088 : 360° : : 485,3772 : 90° 00′ } Angles.
647,1696 : 120 00 }

The fourth terms in both these proportions are found

by taking the ſame parts of the ſecond terms as the third is of the firſt.

This may be called an application of Prob. IV.

PROB. XIV.

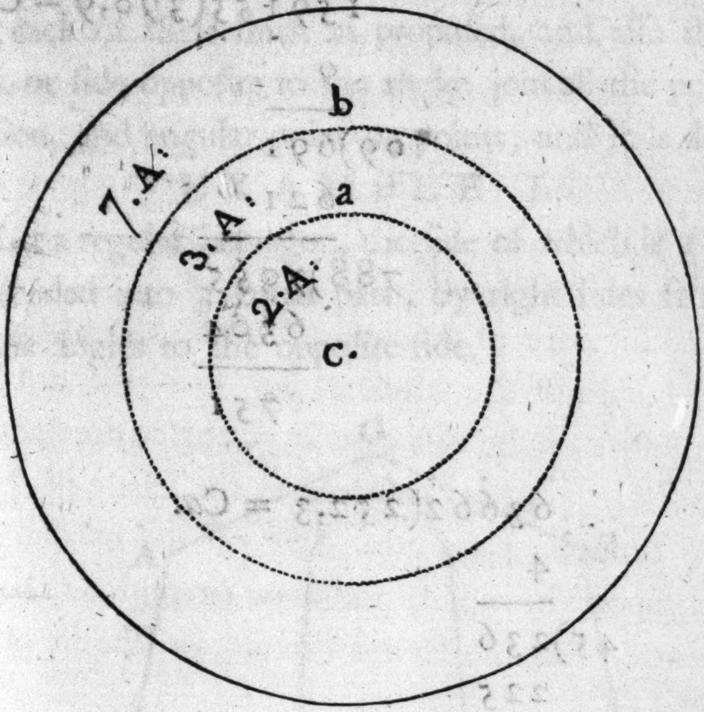
To divide a circle in any ratio propoſed by concentric circumferences.

RULE.

Find the parts of the radius thus, As the whole area, to the ſquare of the whole radius, ſo is the firſt ſhare, to the ſquare of the firſt part, from the centre; and ſo is the ſum of the firſt two ſhares, to the ſquare of the ſum of the firſt two parts &c. Lay off the meaſures thus found, upon the radius, and draw circumferences about the centre, with the ſeveral extents, or lengths of theſe meaſures.

EXAMPLE.

Let a circle, containing 12 acres, be divided, as above, into ſhares of 2, 3, and 7 acres.




```
                    . . . . 2)
,7854)1200000,0000(1527884 ( 1236 = Diameter.
       . . . . . . . . . .
      7854          22)52          618 = Radius.
      41460             44
      39270        243)878
       21900             729
       15708            14984
       61920
       54978
        69420
        62832      4)1527884
         65880        381971 = Square of the Radius.
         62832
          30480
```

```
                    2 : 63662
12 : 381971 ::
                    3 : 95493
                        ------
                        159155(398,9 = Cb.
                        9
                        --
                     69)691
                        621
                        ----
                    788)7055
                        6304
                        ----
                         751

               63662(252,3 = Ca.
               4
               --
            45)236
               225
               ---
           502(1162
               1004
               ----
                158
```

This Problem may be called an application of the Vth.

PROB. XV.

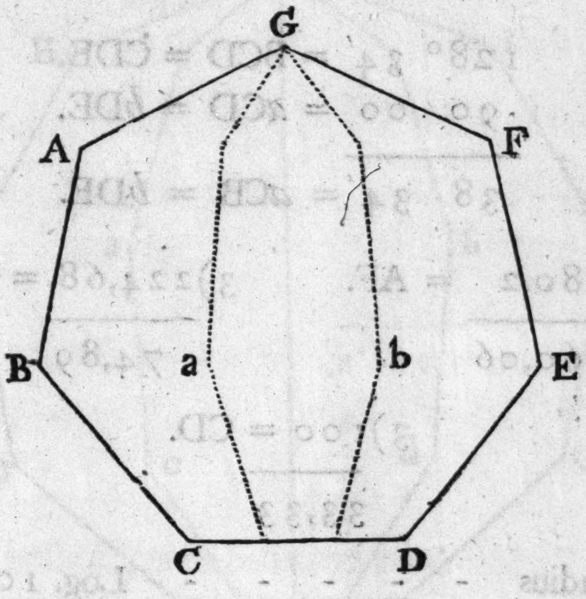
To divide a regular polygon, in any ratio, by right lines from one angle, or one side, to the opposite angle or side.

RULE.

Join all the angles with right lines opposite to the angle that is to be divided, or parallel to the side: divide each of these lines, as proposed, and also the two sides, or side opposite to the angle: join all the points of division, and angular point or points; and it is done.

EXAMPLE I.

Let a regular heptagon, the side of which is 1 chain, be divided into 3 equal parts, by right lines from one of the angles to the opposite side.



Tabular number = 1.15228
100

Radius = 115,228

7)360° 00′

2)51 26

180 00

128 34 = Angle of the heptagon = AGF.

25 43 = $\frac{1}{2}$ of its fupplement = GAF = AFG.

As fine of GAF = 25° 43′ - Log. 9.63741

To GF = 100 - - - L. 2.

So is fine of AGF = 128° 34′ - L. 9.89314

To AF = 180,2 - - - - - L. 2.25573

128° 34′ = BCD = CDE.

90 00 = *a*CD = *b*DE.

38 34 = *a*CB = *b*DE.

3)180,2 = AF.

60,06

3)224,68 = BE.

74,89

3)100 = CD.

33,33

As radius - - - - - Log. 10.

To BC = 100 - - - - - L. 2.

So is fine of *a*CB = 38,34 - L. 9.79478

To *a*B = *b*E = 62,34 - - L. 1.79478

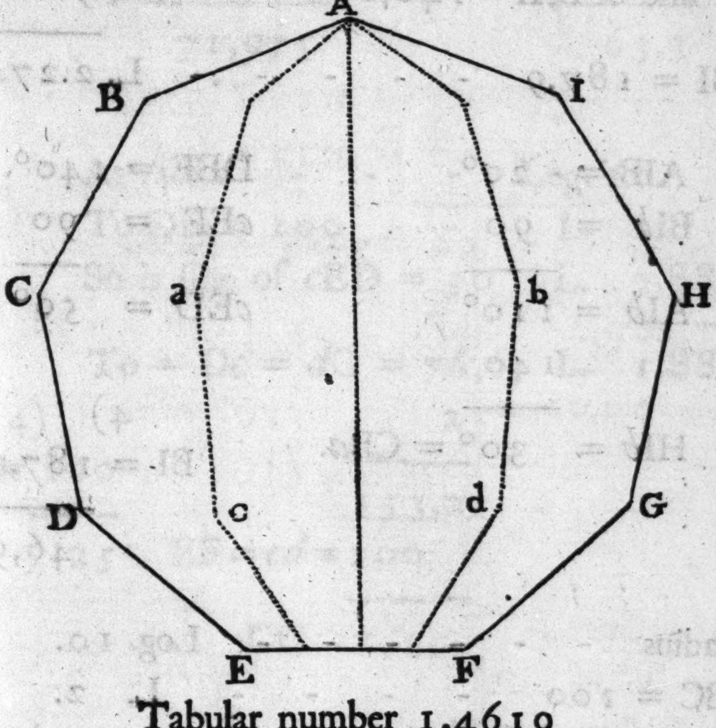
$$\begin{array}{r} 62,34 \\ 2 \\ \hline 124,68 \end{array}$$

$$CD = ab = 100$$

$$BE = 224,68$$

EXAMPLE II.

Let a regular enneagon, the side 100 feet, be divided into 4 equal parts by right lines, as above.



Tabular number 1,4619

$$\begin{array}{r} 1,4619 \\ 100 \\ \hline \end{array}$$

Radius = 146,19

$$
\begin{array}{rl}
9)360^\circ & 00' \\
\hline
2)40 & 00 \\
180 & 00 \\
\hline
140 & 00 = \text{Angle of the Enneagon} = BAI. \\
20 & 00 = \tfrac{1}{2} \text{ of its supplement} = ABI = AIB.
\end{array}
$$

As sine of AIB = 20° 00′ - - Log. 9.53405
To AB = 100 - - L. 2.
So is sine of BAI = 140,00 - - L. 9.80807

To BI = 187,9 - - - - - L. 2.27402

$$
\begin{array}{ll}
AIB = 20^\circ & DEF = 140^\circ \\
BIb = \underline{90} & cEF = \underline{90} \\
AIb = 110^\circ & cED = 50^\circ \\
\quad\quad \underline{140} & \\
HIb = 30^\circ = CBa. & 4)BI = \underline{187,9} \\
 & \quad 46,975
\end{array}
$$

As radius - - - - - - Log. 10.
To BC = 100 - - - - L. 2.
So is sine of CBa = 30 - - L. 9.69897

To Ca = bH = 50 - - - - - L. 1.69897

$$\begin{array}{r} 50 \\ 2 \\ \hline 100 \\ ab = BI = 187,9 \\ \hline CH = 287,9 \end{array}$$

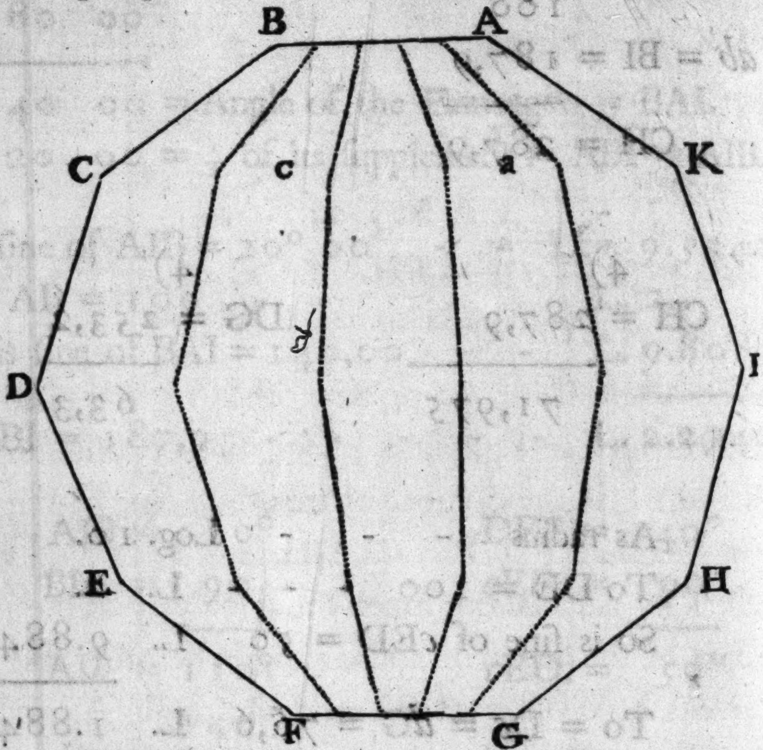
$$\begin{array}{rl} 4) & CH = 287,9 \\ \hline & 71,975 \end{array} \qquad \begin{array}{rl} 4) & DG = 253,2 \\ \hline & 63,3 \end{array}$$

As radius - - -	Log.	10.
To DE = 100 - - -	L.	2.
So is sine of *c*ED = 50	L.	9.88425
To = D*c* = *d*G = 76,6	L.	1.88425

$$\begin{array}{rl} 4) & EF = 100 \\ \hline & 25 \end{array} \qquad \begin{array}{r} 2 \\ \hline 153,2 \\ EF = cd = 100 \\ \hline DG = 253,2 \end{array}$$

EXAMPLE III.

Let a regular decagon, the side 96 links, be divided into 5 equal parts by right lines joining two opposite sides.



Tabular Number = 1,618
96
9708
14562
Radius = 155,328
2
Diam. = DI = 310,656

10)360
2)36
180
144 = Angle ABC.
AaK = 90 = BcK.
54 = CBc.

5)96
19,2

5)310,656
62,131

As radius - - - - - -	Log.	10.
To the side = 96 - - - -	L.	1.98227
So is sine of 54° - - - - -	L.	9.90796
To Cc = 77,67 - - - -	L.	1.89023

$$
\begin{array}{r}
77,67 \\
2 \\
\hline
155,34 \\
5)\ 96 \\
\hline
\text{CK} = 251,34 = \text{EH.} \\
\hline
50,27
\end{array}
$$

The above decagon, or any other regular polygon, may be divided by right lines joining two opposite angles, as A and F; if the number of sides be even, in the following manner, join BK, CI, DH, and EG: find their measures by calculation, much the same way as the joining lines of the heptagon and enneagon before: and join the points of division of BK and the angle A, and of EG and the angle F.

The joining lines are found by Calculation to check the measures on the ground.

PROB. XVI.

To divide a regular polygon in any ratio, by right lines parallel to the sides.

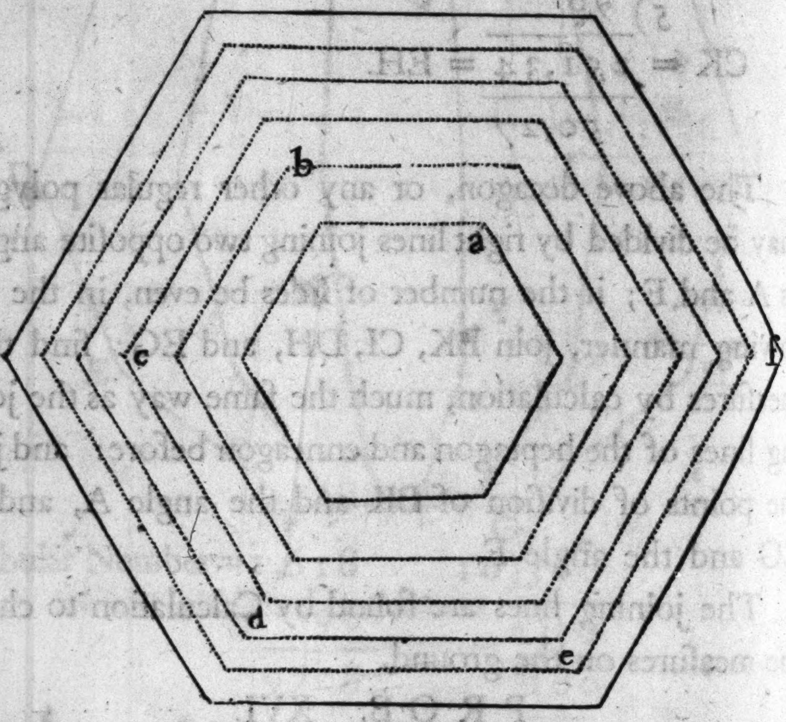
RULE.

Bisect any two adjacent angles, and produce the bisecting lines; they will cut one another in the centre:

draw a radius to every angle; divide each of these radii in the ratio proposed, by the rule of Prob. XIV. and join the points of division all round.

EXAMPLE.

Let it be required to divide a regular hexagon into 6 equal parts, by right lines forming regular hexagons



within it, and let the side of the whole hexagon be 700 feet.

$$\text{Side} = 700 = \text{Radius.}$$
$$\underline{700}$$
$$\text{Square} = 490000$$

Tabular Number = 2,598

490000

23382

6)10392

Content of the whole = 1272020 : 490000 : : 212003,333 : 81666

Sq. of rad. 1ſt Share. Square of 1ſt Part.

212003,333

81666(285,77 = C*a*.

4

48)416

384

565)3266

2825

5707)44166

39949

4217

81666

2

163333(404,14 = C*b*.

804)3333

3216

8081)11733

8081

3652

R

81666
3
245000(495 = Cc.
16
89)850
801
985)4900
4925

81666
4
326666(571,54 = Cd.
25
107)766
749
1141)1766
1141
11425)62566
57125
5441

81666
5
408333(639 = Ce.
36
123)483
369
1269)11433
11421
12

700 = Cf.

Here it may be obſerved, that the two laſt Problems are alſo applications of the IV. V. and VI. Thus, by the help of theſe three Problems, any field of any figure whatſoever, regular or irregular, may be ſeveral ways divided.

And now I have pretty fully explained, as I think, the whole of the land-meaſurer's work, except the Planning Part. Before I proceed to that, give me leave to offer a few cautions to the Young Surveyor, or general directions, reſpecting all theſe four parts.

I. Never begin any field work, either ſurveying, laying out, or dividing, till you have a true repreſentation of the figure of the ground in your head: and never leave the field, till all that can be done there is finiſhed.

II. Truſt more to the meaſures of lines, than angles; and uſe all the checks and proofs you can.

III. Never overlook an error, becauſe it is ſmall; for the ſum of many ſmall ones may amount to one great error.

IV. If you can do one thing ſeveral ways, and all good, chuſe the ſhorteſt and eaſieſt; but never prefer the eaſieſt to the moſt exact, if there be the leaſt difference.

PART V.

PLANNING.

UNDER this title my design is, to comprehend all the necessary and proper rules for making a fair draught or representation of the figure and surface of any field, farm &c. after it is surveyed, and its contents found; of any ground that is laid out, or divided; and of the remarkable things, such as towns, houses, woods or planting, rivers, lakes or ponds, hills, rocks, hollows, remains of antiquity &c. that may be seen within or near about the ground: and for doing the same, when the content is not required; but the plan or map only, as of a parish, county &c. expressing the true figure, situation, and proportion of the whole and all the particulars, so as they may be easily distinguished, their magnitudes estimated, and distances computed; but not to be measured for finding the contents. If it were possible to make a plan exact enough for this purpose, which I doubt very much, I don't know the least necessity for any such thing, nor any ease or advantage of any kind to be gained by it. I mean a plan made by the dimensions taken in the field; and shall except a plain table draught, from a scale of 200 links in an inch, at least, which may come pretty near to the true con-

tent: but I know no other exception. Neither is the Planning, which I propoſe to explain, deſigned for a landſkip or perſpective view: for I think a land-meaſurer is not obliged to be an architect and a painter, nor a compleat geographer neither; for it is not maps of kingdoms &c. that I intend.

The inſtruments proper for Planning may be theſe: I. Scale and Compaſſes. II. The Plain Table. And, III. The Sector. I need not ſpeak of Rulers, Pens, Pencils &c.

I. The Scale and Compaſſes. A long deſcription of theſe inſtruments, or their uſe, ſhould ſeem needleſs to any one who has been taught Plain Trigonometry. I ſhall therefore only obſerve, that the ſcale moſt proper for planning is a Diagonal one, ſuch as that upon the common gunters, having one inch diagonally divided into 100 equal parts, and half an inch divided the ſame way; which, if you pleaſe, you may call the Long Scale and the Short Scale. If the length be 20 or more inches, it will be the better. You ſhould have one pair of compaſſes to extend the whole length of the ſcale, and another pair of ſmall ones, with fine ſharp points, for meaſuring or making angles.

If the meaſures of the lines of a plan are taken from the long ſcale, the plan is ſaid to be laid down from a ſcale of 100 in the inch; if from the ſhort ſcale, it

is said to be laid down from a scale of 200 in the inch: if double the measures are taken from the long scale, it is called a scale of 50 in the inch; if one third, it is called 300; one fifth, is 500 in the inch &c. one fourth of the measures from the long scale, or one half from the short, is 400; one third from the short scale is 600 in the inch: $\frac{1}{10}$ of the measures from the long scale is 1000; from the short scale, 2000 in the inch &c. so that by such a scale as this, you may lay down the measures of almost any determined length you please, in proportion to one inch.

II. The Plain Table. This instrument is made of wood, of a rectangular form, having a ledge or frame, to keep a sheet of paper close and tight upon it: the frame goes off and on, as you have occasion to take off, or put on the paper: it is commonly graduated, that you may measure an angle with it, when you have no other instrument at hand for that purpose. The table may be made of three pieces, to be put together when you use it, the frame helping to keep the seams close, and to be taken asunder for convenience in carrying. It is mounted upon a three-legged staff &c. as the graphometer. The use of it is, to draw, upon the spot, a similar figure, or first draught of the field which you are surveying: for this purpose, you have an index with two high sights, to ly upon the paper any way required, and

you may have a ſcale upon it, or one ſide of the frame; or you may take a ſcale and compaſſes with you.

III. The Sector conſiſts of two equal legs, the ſame way divided with regard to one another, by ſeveral lines, all drawn from the ſame centre, which is the middle point of a joint about which the legs move and open to any extent required: theſe lines are all drawn and divided the ſame way on each of the legs, ſo as to run out all of them in pairs, as two lines of lines, two of chords, two of tangents &c. Of all theſe lines, the only pair for our purpoſe here is that of the lines of chords, which, if the legs be 8 inches long at the leaſt, will ſerve, much more exactly than any other inſtrument commonly uſed, for making or meaſuring angles. A ſector may be made for this uſe only, with each of the legs one foot long, having the lines of chords on one face, diagonally divided; and on the other a diagonal ſcale of the length of two feet, when the legs are laid quite back, and made to join in one ſtraight piece, from one end to the other.

Before I proceed to the particular rules for forming the firſt draught of a field or ſurvey, it may not be amiſs here, to diſtinguiſh the uſes of the eye-draught, rough draught, and this now mentioned firſt draught of a plan.

The only uſe of the eye-draught is to direct the meaſures when you ſurvey by the chain only; and there-

ſore very little exactneſs is required in it; if the lines ly all the ſame way as in the field, their proportion to one another is little regarded.

The uſe of the rough draught is to direct you in finding the contents of ſurveys by the graphometer and in dividing; the proportion of the lines and angles to one another is ſo far to be obſerved, as not to take a great one for a ſmaller, or a leſſer for a greater; but the exact meaſures of either are not required.

The uſe of the firſt draught here, is to make the finiſhed plan by it; and therefore the greateſt exactneſs poſſible, in every particular, is abſolutely neceſſary: tho' not ſo clean and beautiful, it muſt have all the truth and exactneſs that is intended in the plan.

PROB. I.

To lay down a ſtraight line of any meaſure required, by the ſcale.

RULE.

Reduce the meaſure to the ſcale which you intend to uſe for the plan: place one foot of the compaſſes in the point where the perpendicular of the hundreds interſects the parallel of the units, and extend till the other foot falls into the point where the diagonal from the tens interſects the ſame parallel: with this extent ſweep a faint arch from the point the line is to be drawn from, and draw a right line from this point to any point of

the arch proper for your purpoſe. If the direction of the line is given, the arch is needleſs; as on the plain table, or the firſt line of any draught.

EXAMPLE.

Suppoſe 2568 links to be laid down from a ſcale of 400 in the inch.

Half of 2568 is 1284. Fix one foot in the perpendicular under 12 of the ſhort ſcale, where the 4th parallel cuts it, and extend till the other foot falls into the point where the diagonal from 8 cuts that parallel, and you have the extent required.

If 0 be in the units place, take the extent from the hundreds to the tens upon the top line.

PROB. II.

To find the meaſure of any right line in a plan, by the ſcale.

RULE.

Take the extent of the line in the compaſſes: place one foot on the top line, at ſuch of the hundreds as will bring the other foot into the inch or half inch that is diagonally divided; move both the feet downward till one of them fall into the point where the diagonal from the neareſt ten cuts the ſame parallel where the other foot ſtands, and you have the hundreds, tens, and units required: then reduce by the proportion of the ſcale to one inch.

EXAMPLE.

Suppoſe one foot of the compaſſes ſtanding in the point of interſection of the diagonal from 8 and the 4th parallel, when the other is in the interſection of the 12th hundred and the ſame parallel, upon the ſhort ſcale, and the ſcale of the plan 400 in the inch; the extent will be then 1284, which doubled will make the meaſure of the line required 2568 links.

If the line is too long for your ſcale, it muſt be laid down, or meaſured firſt one part, then another &c. but it is not eaſy, nor ſure work, to make any figure that hath more than one of ſuch lines, and therefore your ſcale ſhould be a long one.

PROB. III.

To make an angle nearly of any meaſure required, by the ſector.

RULE I. By one line of chords.

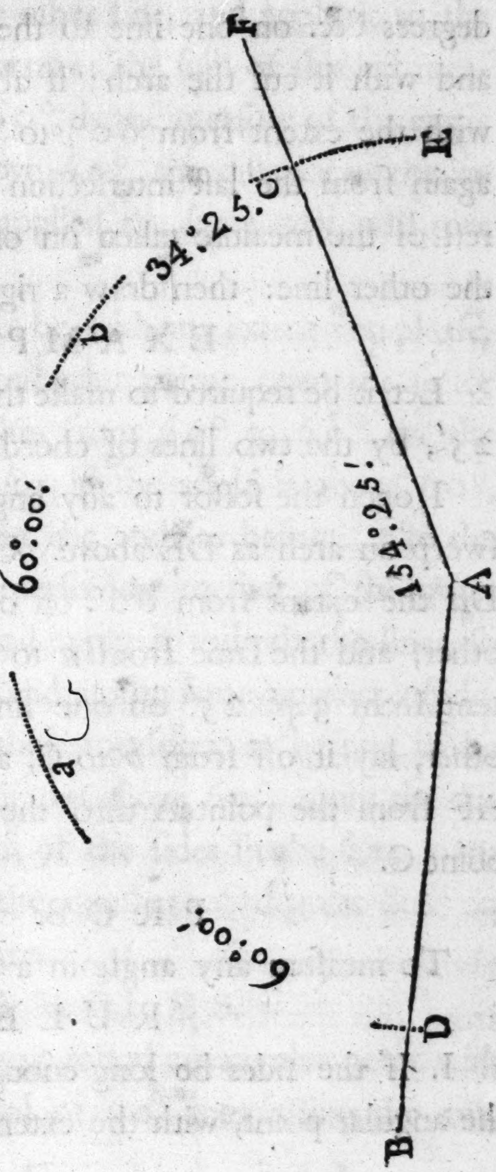
With the extent from the centre to 60° 00′ ſweep an arch, large enough to hold the meaſure, and cutting the line, from the angular point: then, if the meaſure is not above 60°, take the extent of it from the centre, and cut the arch, from the interſection of the line; if above 60°, cut the arch with the extent of 60°, and from this interſection, cut it again, with the extent of the reſt of the meaſure; if above 120°, cut it twice with the extent of 60°, and a third time with the reſt of the

measure: in every case, take the odd minutes as near as you can. Draw a right line from the point thro' the last intersection.

EXAMPLE.

If it were required to make an angle of 154° 25', with any given right line, as AB, and at the point A.

With the extent of 60° in the compasses, and one foot in A, I sweep the arch DE: with one foot in D, and the same extent, I cut the arch in *a* and *b*: the last intersection lays off 120°, with one foot in *b*, and the extent of 34° 25'. I cut the arch again in C, then I draw the right line ACF which forms the angle required with AB.



RULE II. By the two lines.

Open the ſector to any angle you pleaſe: with the extent from 60° to 60°, ſweep an arch as before: if the meaſure is not above 60°, take the extent from the degrees &c. on one line to the ſame, on the other line, and with it cut the arch; if above 60°, do as before with the extent from 60° to 60°, and cut the arch again from the laſt interſection with the extent of the reſt of the meaſure taken on one line to the ſame on the other line: then draw a right line as above.

EXAMPLE.

Let it be required to make the above angle of 154° 25′, by the two lines of chords.

I open the ſector to any angle, and keeping it ſo, I ſweep an arch as DE above. Upon this arch I lay off D*a* the extent from 60° on one line to 60° on the other, and the ſame from *a* to *b*; then I take the extent from 34° 25′ on one line, to 34° 25′ on the other, lay it off from *b* to C, and draw the right line AF from the point A thro' the laſt interſection at the point C.

PROB. IV.

To meaſure any angle in a plan, by the ſector.

RULE.

I. If the ſides be long enough, ſweep an arch from the angular point, with the extent of the chord of 60°

on one of the lines, interſecting them both: if the angle is more than 60°, cut this arch once or twice from the point of interſection of one ſide, with the ſame extent: take the diſtance from the laſt interſection of the arch to the interſection of the other ſide, and apply it to the line of chords from the centre: the ſum of this laſt meaſure, and once or twice 60°, is the meaſure of the angle required: but if not above 60°, the diſtance of the interſections of the ſides, applied the ſame way, will give the meaſure required.

II. Whatever the ſides be, with any extent you pleaſe, interſect them from the angular point: open the ſector till this extent juſt reaches from 60° to 60° on the two lines, and keep it ſo: if the angle is above 60°, with the ſame extent, cut the arch as before: take the diſtance from the laſt interſection to that of the other ſide in the compaſſes, and apply it to both the lines, ſo as both the feet may ſtand at the ſame number of degrees &c. the ſum of theſe meaſures, as before, is the meaſure required: but if not above 60°, apply the diſtance of the interſections of the ſides in the ſame manner to the two lines; the number of degrees &c. on them both is the meaſure.

EXAMPLES.

Suppoſe an arch, ſwept round an angular point with the extent of the chord of 60° interſecting the two

sides, contains that extent, and $37° 15'$ more, from the centre; and the measure of that angle will be $97° 15$.

Suppose again, an arch swept, as above, with any extent, and the distance of the intersections of the sides to reach from $56° 40'$ on one line to $56° 40'$ on the other, that angle will contain $56° 40'$.

Suppose once more, the sides of an angle intersected with any extent, and the arch to contain that extent twice, and the extent from $24° 10'$ to $24° 10'$, that angle will be $144° 10'$.

PROB. V.

To form, or make the first draught of any run-rig measure.

RULE.

Lay down a faint line equal to the right line from pole to pole: put a mark at the end of each of the measures set down in the field-book under L, in the same order as they stand there: from each of these marks raise perpendiculars equal to the measures under B, on both sides, and mark the tops of them: but produce such as have measures standing without, till they are equal to the sum of both, and mark the end of the part without, next to the line: draw a strong prick'd line with a steady hand, joining all these points which you have marked, along both sides, and at both the ends; and you have done.

EXAMPLE.

Let it be required to form the first draught of D. BLACK's rig, p. 55. from a scale of 100 links in an inch. See Plate V.

Lay down, by Prob. I. a right line equal to 1485: put a point or mark at the end of 100, 200, 260, 300, 400 &c. from the first mark, raise a perpendicular to the line, equal to 42 to the right hand; from the second mark, another the same way, of 28, and one to the left of 16; from the third, one to the right of 24, and one to the left of 20; from the fourth mark, 17 to the right, and 27 to the left; from the seventh, 43 to the left; from the ninth mark, 42 to the left, but a point at the end of 6 next to the line; from the tenth, 56, but a point again at the end of 19 from the line &c. Join all these points as you may observe in the draught: write the name of the rig &c.

PROB. VI.

To make a first draught of a field surveyed by the chain only.

RULE.

Take first one sheet of common paper; upon this lay down the several triangles that make up the right-lined part, joining them all in their proper places, as directed by the eye-draught, and let the several measures be all taken from a small scale, as 1000 or 2000 in

an inch. By this rough draught you will ſee what will be the ſize and figure of your plan, from the larger ſcale, you deſign to uſe. Take one, or more ſheets, handſomely joined together, of large paper, and upon this lay down the right-lined part from your intended ſcale; all the ſides faint lines, or drawn by one point of the compaſſes: ſet off the off-ſet perpendiculars in their true places, as the breadths in laſt Problem, and draw the boundary thro' their tops; let it be a ſtrong pricked line for open grounds, and a black line for incloſures: then repreſent all the remarkable things in their proper places: but take care that all the lines whatſoever be taken from the ſame ſcale.

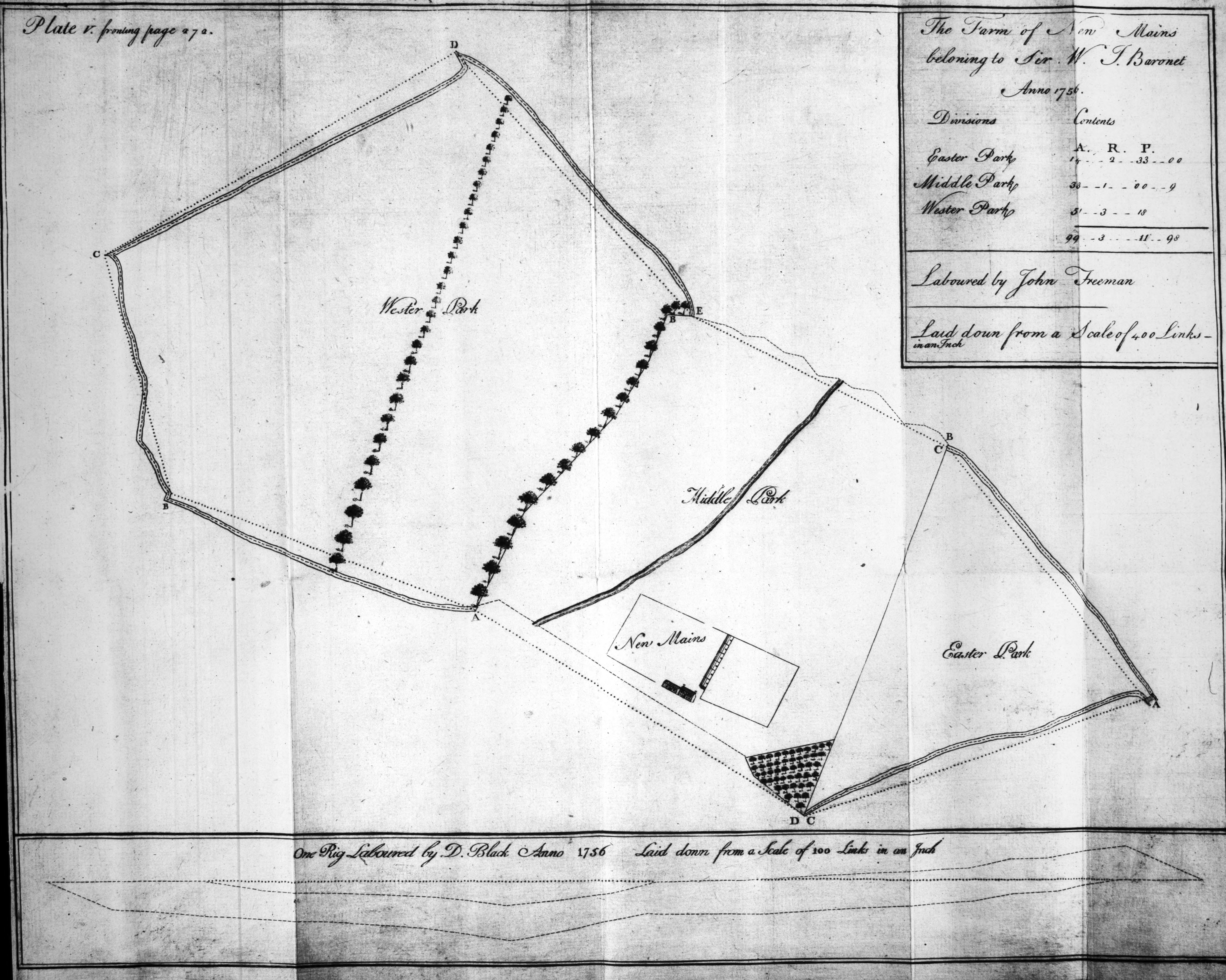
EXAMPLE.

Let a firſt draught of New-Mains Farm, p. 60, be required, from a ſcale of 400 links in an inch.

See Plate V.

Half the meaſures of the ſides &c. taken from the ſhort ſcale will be 400 in the inch. Make therefore a triangle, whoſe ſides are 838 = AB, 1030 = BC, and 934 = CA: this triangle will repreſent the right-lined part of the Eaſter-Park. Mark the ends of the ſeveral meaſures ſet down in the middle column of the field-book, under Diſt. from A towards B, and from C towards A: from theſe points or marks raiſe the off-ſet perpendiculars taken from the ſame ſcale, thoſe in the

Plate V. fronting page 272.



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ight hand column (under off-ſets) to the outſide, and thoſe n the left hand column to the inſide: make repreſentatiоns of the things remarked in their true places, as direced by the field-book: draw the boundary, as you ſee; nd you have done with this part of the farm. After he ſame manner, make the triangle ACD, having CD he ſame with BC in the Eaſter Park, and the triangle ABC having AC common to it and ACD: do with the ff-ſet perpendiculars, remarks and boundary, as before, nd you have the Middle Park. Make the triangle AEB, aving AE the ſame with AB in the Middle Park, the iangle DBE having BE common to it and AEB, and he triangle CBD having BD common to it and DBE: o with the off-ſet perpendiculars &c. as before, and ou have the Weſter Park, and the whole farm truly reреſented.

PROB. VII.

To ſurvey from one ſtation, where all the corners an be ſeen, or lines to them found, and make the firſt raught of the field, by the plain table.

RULE.

Set the inſtrument, mounted and covered with a ſheet f clean paper, upon the ſtation: level it ſo as you can ee all the poles or marks when you look over it, and hen ſcrew faſt. Chuſe a point on the paper to repreſent your ſtation, and ſo as you can contain the whole

figure of the field upon it, from your intended scale: lay the central edge of the index on the point, and keeping it there, turn the index about, till you see one pole thro' the sights; there draw a faint line from the point by the edge of it: measure from the table to that pole, and from the next pole to the table again; lay off the first measure from the station point upon the faint line: turn the index, as before, to that next pole, draw another faint line from the point by the edge, and upon this line lay off the second measure. Proceed in this manner, measuring one line from the table, and another to it again, and laying off these measures from the station point upon faint lines, drawn as before directed, till you have all the lines from the station to the several corners measured, and laid off upon the paper. Join the ends of all these faint lines with right lines representing the boundary; and you have done.

See an example in the following eye-draught.

Note I. The field must not be very large, neither must there be any off-sets upon the boundary, when you apply this Problem. When the field is large, it may happen to be tedious; and when there are off-sets, it is impracticable. Another method is to be taken, which shall be shown hereafter, when the bounding lines are crooked. See Prob. X.

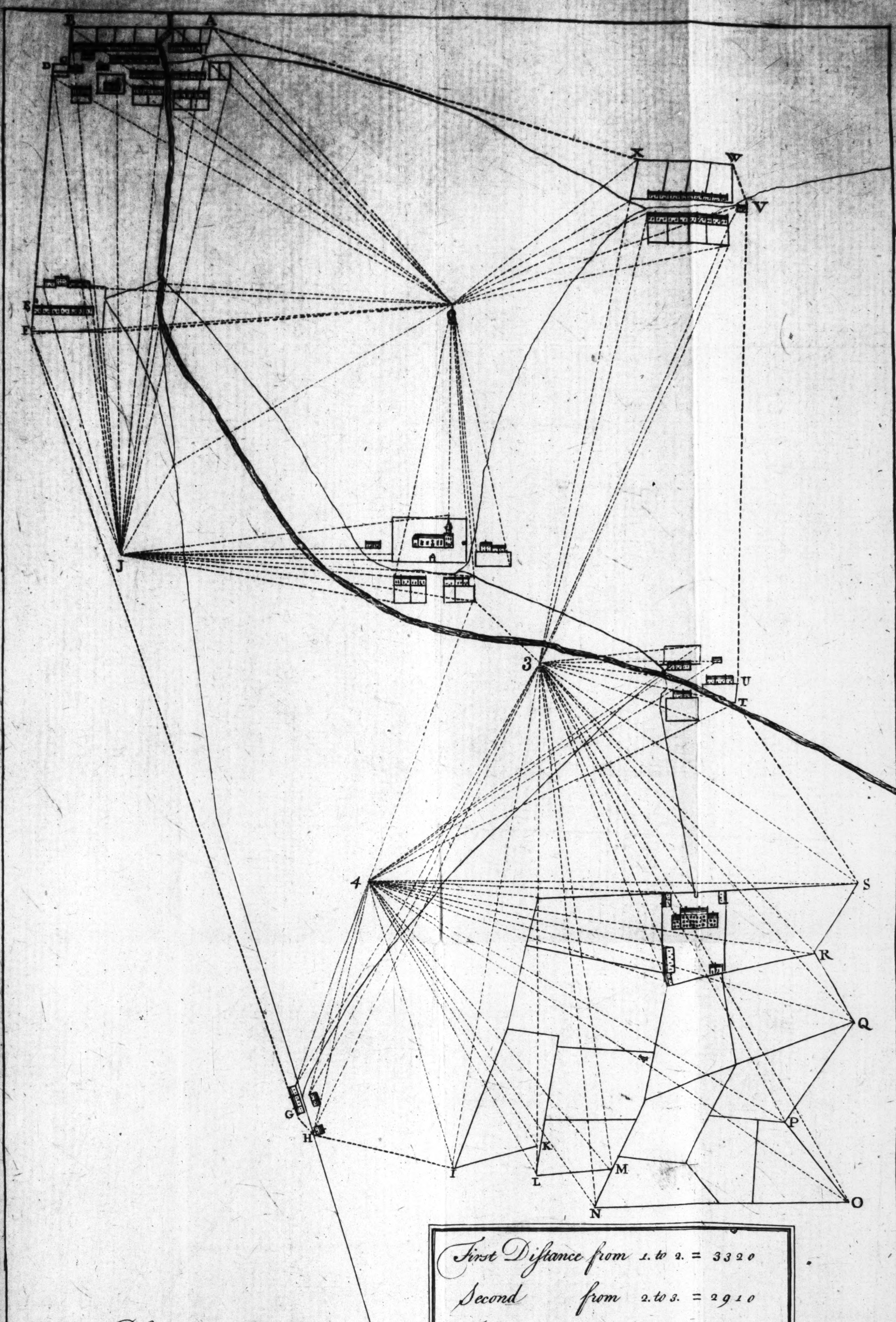

Plate VI. fronting page 276.

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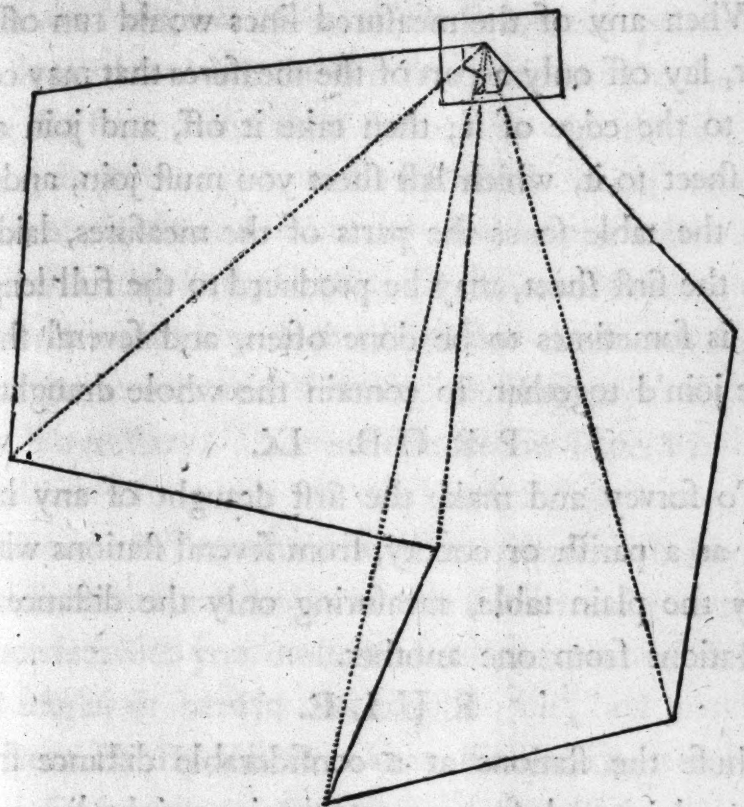
Eye-Draught.



Note II. A large field may be ſurvey'd and plann'd much after the ſame manner from two, three, or more ſtations; if you take in the firſt ſtation with the corners, to which you meaſure from the ſecond, the ſecond with thoſe you meaſure to from the third &c. and then join all the corners, as before: but ſtill there muſt be off-ſets upon none of the ſides thus found.

PROB. VIII.

To ſhift the paper upon the plain table.

RULE.

When any of the meaſured lines would run off th paper, lay off only a part of the meaſures that may com near to the edge of it; then take it off, and join ano ther ſheet to it, which laſt ſheet you muſt join, and pu upon the table ſo as the parts of the meaſures, laid of upon the firſt ſheet, may be produced to the full length This is ſometimes to be done often, and ſeveral ſheet to be join'd together, to contain the whole draught.

PROB. IX.

To ſurvey and make the firſt draught of any larg tract, as a pariſh or county, from ſeveral ſtations withi it, by the plain table, meaſuring only the diſtances o the ſtations from one another.

RULE.

Chuſe the ſtations at a conſiderable diſtance from one another, and ſo as not to be in a right line wit any of the angles or corners, or marks you deſign to uſ as ſuch. Having ſet the table at the firſt ſtation, leve it ſo as you can ſee the ſecond and all the viſible angle over it, then ſcrew faſt. Direct the index, as in Prob. VII to the ſecond ſtation, and all the angles, one after ano ther, and draw faint lines by the edge of it, as before Meaſure to the ſecond ſtation, and there direct the in dex, as before, to the firſt ſtation, and each of the ſam angles, drawing faint lines &c. theſe faint lines, thu

drawn, will interſect one another in points repreſenting the ſeveral angles. Join all theſe angular points, and you have done with the firſt two ſtations. Do the ſame with the ſecond and third ſtations, as with the firſt two, taking in two angles, near the third ſtation, and laid down at the ſecond, with the other angles which come in view at the ſecond and third ſtations. Proceed after the ſame manner with the third and the fourth ſtation, with the fourth and fifth &c. ſhifting paper as often as it is neceſſary. See an example in Plate VI.

Note I. When the caſe of two ſtations being in a right line with any angle, or one ſtation and two angles, is unavoidable, you muſt meaſure from one of them to that angle, or betwixt the angles.

Note II. By this Problem alſo you may ſurvey a ſmall field without off-ſets, from two ſtations without it; when it is not allowed to go into it, for ſtanding corn, or any other reaſon.

PROB. X.

To ſurvey and make the firſt draught of any field or tract of ground whatſoever, by the plain table, going round it.

RULE.

Place the table at one angle or corner, and meaſure from another to this angle, taking the off-ſets, and noting

down the remarks all the way as you go on: when you come to the table, level it to the angle from which you came, and to the next on the other ſide of it, and ſcrew faſt; chuſe a point on the paper to repreſent th angle where you ſtand, and keeping the central edge o the index upon it, turn the ſights to the firſt angle, dra a faint line by the edge, and lay off the meaſured ſid upon it: then keeping the table faſt, and the index o the ſtation point, turn the ſights to the next angle, an draw another faint line as before: meaſure to that ne angle, and lay off the meaſure upon the faint line. A this third angle, having levelled the table as before, la the index upon this laſt meaſured line, and keeping there, turn the table about on the head of the ſtaff, t the ſights are on the ſecond angle, then ſcrew faſt gain: keep the central edge on the end of the laſt me ſured line, which now repreſents your preſent ſtatio and turn the ſights to the fourth angle; meaſure &c. before, to the fourth angle. Go on in this way, you come about to the firſt angle. If your draught clo exactly there, all is right; but if not, there has been ſo error either in meaſuring, or laying off the meaſures: find out which, the only ſure way is to prove all lines by meaſuring back again; as it has not been covered till the work is done. But an error may be covered and corrected in the time of working by

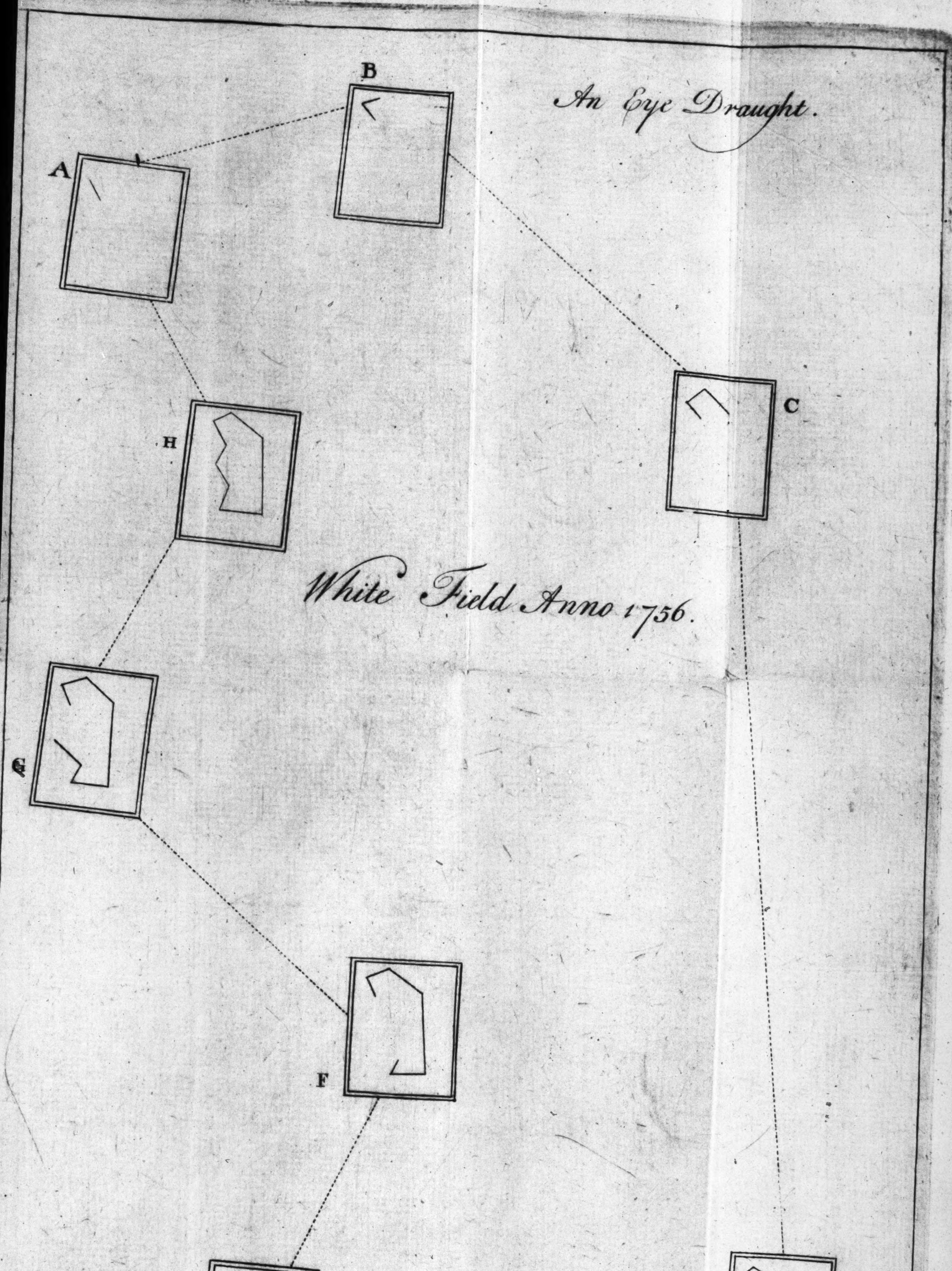


Plate VII fronting page 278.

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following Problem. See an example in the Eye-Draught, Plate VII. where you may obſerve, that the plain table is always parallel to itſelf at every one of the ſeveral ſtations.

PROB. XI.

To prove the work of laſt Problem in the time of working.

RULE.

At the firſt angle, where the table is ſet, if you can ſee any remarkable thing, or ſet up a pole that will be viſible from every angle, draw a faint line towards it, as before directed; at the next angle do the ſame: theſe lines will interſect in a point of the draught repreſenting its place on the ground. At each of the other corners or angles do the ſame again. If all theſe lines interſect in the ſame point, you are right, and where-ever one of them fails, the laſt meaſured ſide is wrong, or laid off wrong.

If the thing you obſerve inſtead of a pole be not as exactly diſtinguiſhable, as a tree, or ſtanding ſtone; it is better to ſet the pole.

Note. By this and the X. Problem you may ſurvey for the content, meaſuring the neceſſary dimenſions, but let the ſcale be at leaſt 200 in the inch.

PROB. XII.

To ſurvey, for a plan only, from ſeveral ſtations, by the graphometer, meaſuring only the diſtances betwixt the ſtations.

RULE.

Take all the angles formed, as on the plain table draughts, by the right lines from the ſeveral ſtations to the corners, obſerving the ſame caution in the choice of your ſtations &c.

Take ſuch an example as that of Prob. IX.

Station M Eaſt from N.		
	AMN	121° 20′
	BMN	94 30
	CMN	86 10
	LMN	31 15
	KMN	21 00
	MN	12000
Station N.	ANM	28 05
	BNM	29 50
	CNM	67 30
	LNM	36 45
	KNM	106 25
	KNO	167 55
	CNO	15 48
	INO	88 55
	HNO	47 50
	DNO	5 28
	ENO	10 10
	NO	8000

Station O.		
	KON	4° 40′
	CON	132 18
	ION	69 10
	HON	79 35
	DON	171 10
	EON	153 50
	HOP	56 15
	EOP	16 52
	FOP	22 30
	GOP	63 40
	OP	10000
Station P.	HPO	50 38
	EPO	144 16
	FPO	118 20
	GPO	90 00

The ſtations may be choſen either within or without, or at the angles. If they are all within, you may ſurvey any tract of ground, how large ſoever; but it is for the plan only: if without, the ſurvey is alſo for the plan only; but the field muſt not be large: if the ſtations are at the angles, the caſe is the ſame as in Prob. XXIX. of Part I. and the ſurvey may ſerve both for the content and the plan.

PROB. XIII.

To make the firſt draught of any ſurvey by the graphometer.

RULE.

Lay down the ſeveral meaſured lines with the angles, in their true places: join the ſeveral corner points, and draw the off-ſet perpendiculars, if any: then draw the boundary, as in Prob. VI.

EXAMPLE.

Let the firſt draught of the Sheep Park, p. 69. be required from a ſcale of 300 in the inch.

Firſt, Divide the ſeveral meaſures of the ſides by 3 to reduce them to your ſcale.

Then lay down the firſt ſide AB (from the long ſcale) and at the point B make the angle ABC, with the ſecond ſide BC: at the point C make the angle BCD, with the third ſide CD &c. Set off the off-ſet perpendiculars in their proper places, as directed by the field-

book, and then draw the boundary. Take all the meaſures from the ſame ſcale from which AB was taken.

See Plate VIII.

PROB. XIV.

To prove the work of a ſurvey round a field, by check lines.

RULE.

Meaſure from one angle to any other, in the field; and betwixt theſe ſame angles in the draught of it: if theſe meaſures very near agree, it is very probable that all the work betwixt theſe angles both in the field and in the draught is right: but if they differ any thing conſiderably, examine both again, to find the error. I ſay, very near, becauſe if there are any inequalities, or unevenneſſes, tho' not great, on the ground betwixt theſe angles, the check lines may differ a little, and yet the work may be right enough. I ſay alſo, very probably right, becauſe two equal errors may happen to balance one another; but this is not a very likely caſe.

This Rule may be applied when you ſurvey by the plain table, and cannot uſe a proof pole, as well as when you ſurvey by the graphometer.

What is ſaid about the remarks in Prob. VI. and referred to the field-book in p. 60, and 61, for New-Mains Farm, is upon the ſuppoſition of their being near the lines you are meaſuring, and ſo as perpendiculars may

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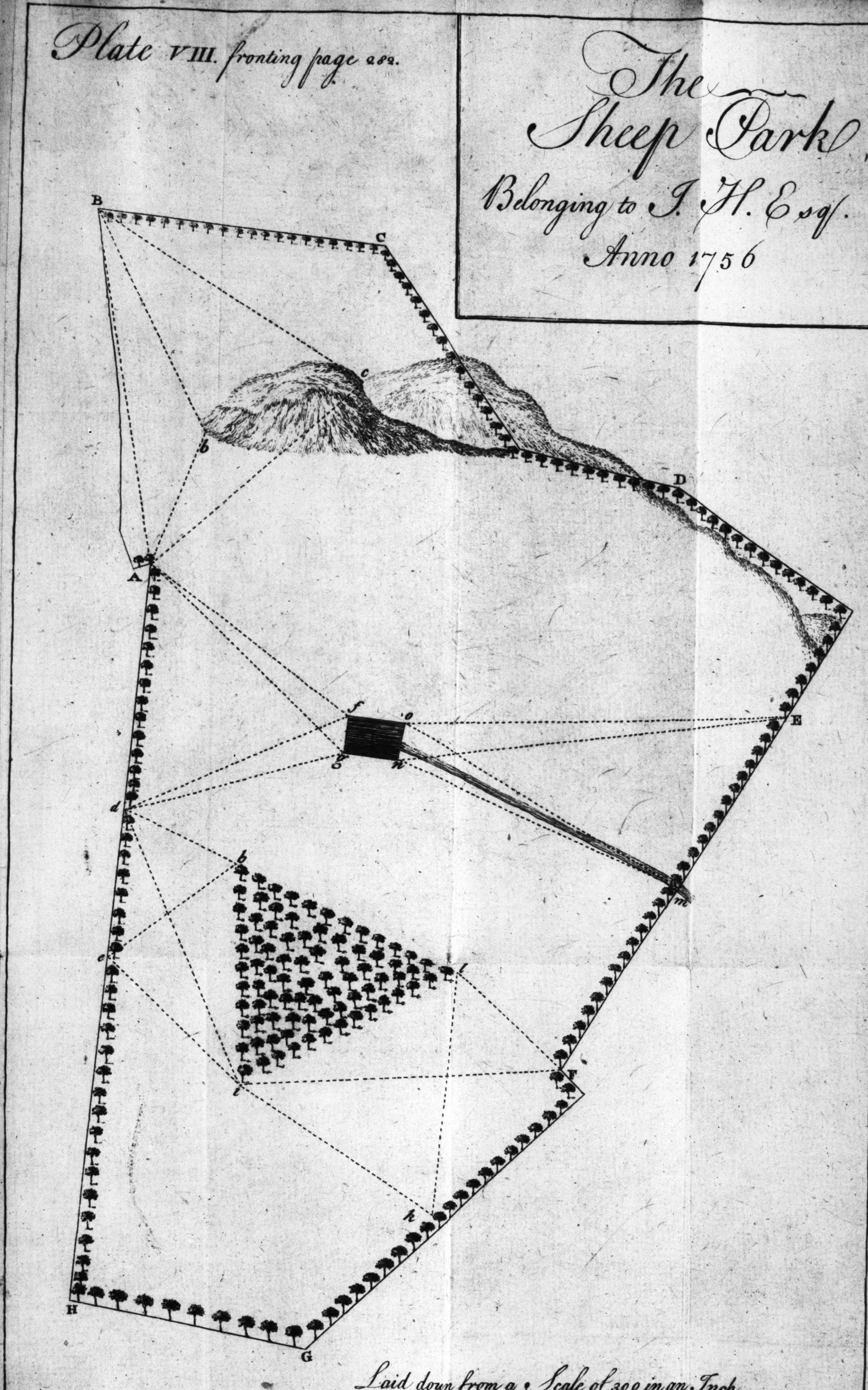
Plate VIII. fronting page 282.



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be eaſily and ſurely ſet off to them: but when it is otherwiſe, the following Problem is to be applied for laying them down more exactly and eaſily.

PROB. XV.

To lay down any remarkable things, as planting, towns, turnings of a river &c. within the bounds of your ſurvey, or near at hand without, when you meaſure round about the ground, or when you run diagonals or check lines; or when you ſurvey any way whatever.

RULE.

If any of the lines go thro' them, mark exactly the part of the line where it enters, and where it comes out again, in the field-book. But if not, chuſe two ſtations upon the line, from whence you can ſee all the angles, turnings &c. that are viſible from that line, marking exactly the ſtations: there draw interſecting lines, as in Prob. IX. to the ſeveral turnings &c. if you uſe the plain table; or take the angles formed by theſe lines, and the line you are upon, at the ſeveral ſtations, if you uſe the graphometer. This may be done at the principal ſtations, if they will ſerve the purpoſe as well. Theſe lines may be meaſured on the ground, when you uſe the chain only, which is as ſure a method as any other. Lay down all theſe meaſures of lines and angles as you ſee done in Plate VIII.

See alſo the field-book in p. 69, and 70,

PROB. XVI.

To enlarge or diminiſh a draught in any ratio required.

RULE.

Meaſure exactly all the ſides, diagonals and off-ſet perpendiculars, by the ſcale of the draught; reduce this ſcale to the ſcale required as from 400 to 200 in the inch, by doubling the meaſures; from 300 to 800, by multiplying by 3, and dividing by 8 &c. then make a new one by Prob. VI. anſwering to this ſcale required. Or, inſtead of meaſuring all theſe lines, you may meaſure only ſuch lines and angles as will determine the reſt, and proceed by Prob. XIII.

Other methods have been propoſed, but I know of none eaſier, nor ſo ſure. Upon trial, this method will be found preferable to thoſe commonly preſcribed by ſquares and rulers &c, whether the draught be large or ſmall.

PROB. XVII.

To finiſh a plan from the firſt draught.

RULE.

Be ſure that all the remarks deſigned for the plan be laid down diſtinctly and truly in the draught, and that every thing be exact and compleat in it. Take paper of the ſame ſize with it, handſomely and neatly joined, if more ſheets than one: lay the draught upon it, faſtened ſo as it may not ſhift any way: then, with a needle, or ſtrong ſharp pin, prick thro' all the corners, turnings

of the lines, angles &c. of the remarks, and ſeveral parts of the long right lines, ſo as when all theſe points are joined, you may have the true and compleat figure of the draught appearing on the paper that is below: but take care not to pierce them both. Take off the draught: join all the points, and make the new paper agree with the other, in all particulars as far as the points direct you, with the help of the draught lying before you; ſo as it may be a fair and clean copy of the firſt draught, as far as that agrees with the figure and ſurface of the ground which the plan is intended to repreſent, without the letters, arches &c. which are only uſeful in the conſtruction of the firſt draught. Then, upon any convenient part, write the name, or general title, and the proprietor's name and titles, the names of the ſeveral parts, and the tenants; alſo the contents, if required: lay down the ſcale of the plan, or refer to it by mentioning what ſcale you have uſed: make a ſmall figure of the mariner's compaſs card, to ſhow the bearing or ſituation of the ground: write alſo the names of the proprietors of the lands, and of theſe lands, that border upon your ground, in their proper places: and draw a clean double line upon each of the four ſides of the paper, incloſing the whole. You may alſo diſtinguiſh the bounding and dividing lines with tranſparent colours, and colour the planting, houſes &c. ſo as they may be the

more eaſily diſtinguiſhed from the reſt: this will give the plan a better look.

Let the names of the ſeveral parts or diviſions ſtand within them, or, if they are but ſmall, ſet marks, ſuch as letters &c. for them, and let theſe marks be explained in a table upon ſome convenient open part of the paper. Do the ſame with the ſmall towns, homeſteads, roads, rivers &c. and if it is a large tract, belonging to many proprietors, do the ſame with the ſeveral lordſhips, mannors &c.

If it ſhould be a tract of ſeveral counties, or a whole kingdom; inſtead of the double lines incloſing the whole, draw a graduated meridian on each ſide, and a line of longitude or meridional diſtance at the top or bottom, with ſeveral parallels to both a-croſs the map. For this purpoſe, the latitude ſhould be obſerved in ſeveral places, and the longitude calculated. But as the making of ſuch maps is the proper buſineſs of a Geographer, and not of a Land-meaſurer, I ſhall ſuppoſe the hint I have now given here ſufficient for my purpoſe: as I have all along ſuppoſed one county to be the utmoſt extent of the ſurvey.

Theſe rules then, I think, may ſerve for planning any actual ſurvey; and therefore I ſhall conclude this fifth and laſt part, with obſerving:

I. Every plan is to be made either from a plain table draught, or a firſt draught formed from a field-book; the

first is the most exact: in both there are two necessary or essential requisites, the titles and scale: without both of which, the plan is good for nothing. Without the general title, or name of the whole tract, and of the several divisions and things represented in it, it is unintelligible; and without the scale, either made or referred to, it is useless, because there is no judging of the distances and magnitudes. From what I have declared as my opinion, before, of the connection betwixt contents and plans, and what I have just now said of the necessity of the scale, I suppose it will not be expected, that I should give any rule for finding what scale was used for the plan, by having the contents and measuring &c.

II. As for the use of the Mariner's compass in surveying, I know of none, save only to take the angle formed by some one of the lines and the Meridian or North and South line, that passes thro' any one of the angles, of which that line is one side, in order to find the bearings or situation of the several parts of the ground, with respect to the meridian and one another: and as this can be done exactly enough for the purpose, by a small pocket compass, or by the shadow of a pole, at mid-day set up perpendicular in the angular point, forming an angle with the line, which angle may be measured by the graphometer; or even by asking the country people, who will point out a meridian line, for the most part, as exactly

as the compaſs, in ſome part or other of the ground: I ſhould think the formality of a Problem, Rule and Example, not only unneceſſary, as ſo little is required, or can be done, but alſo ready to miſlead ſome, by making them lay more ſtreſs upon the thing than it can bear. The meridian thus found may be drawn in its true place in the firſt draught; in any convenient part of which, a ſmall figure of the compaſs card, with its North and South line parallel to it, may be made. This will ſerve all the purpoſes intended, and is all the uſe of the ſea compaſs in meaſuring land. Having mentioned it in the laſt Rule, and it being ſo far uſeful, as well as ornamental, I thought it might be proper to ſay ſomething about it. But if any man will uſe it in taking the neceſſary dimenſions either for the contents or for the plan, it muſt be thought that he has little regard for exactneſs, or that he knows no better. Bad as the Theodolite is, this is yet worſe. That may be amended, but this is incorrigible.

And now, for a concluſion to the whole, give me leave to obſerve, that the beſt method of ſurveying for the contents is by Prob. XXVII. of Part I. that you ſhould never take angles if you can poſſibly do without them: and when you muſt uſe a graduated inſtrument, never truſt the work, without proving it by check lines. This method of Surveying is likewiſe better for a plan, if the ground is not large, than uſing the graphometer;

and in this caſe I would adviſe to calculate all the ſides and diagonals that are not meaſured, and make the firſt draught by Prob. VI. of Part V. when you have been obliged to take the angles, if you would have your draught exact. The beſt method of ſurveying for a Plan, in my humble opinion, is by the plain table, when it can be uſed; but when the ground is large, and the plain table cannot be uſed, then you muſt uſe a graduated inſtrument, to ſave the trouble of meaſuring every line by the chain, and alſo to do the work when the chain cannot be uſed for every line, which is very frequently the caſe. A good graduated inſtrument, therefore, is abſolutely neceſſary in ſurveying, tho' not in planning; becauſe all the lines, that cannot be meaſured, can be calculated. Working by lines and angles may be quicker than by lines only, but never can be ſurer. In every part of a Land-meaſurer's work, lines are more to be truſted than angles.

APPENDIX.

OF INSTRUMENTS FOR THE LAND-MEASURER'S USE.

IT is very proper, and even often necessary, for every Land-measurer to know so much of the making or constructing his instruments, especially of the graduated kind, as may enable him, not only to judge of their use and goodness, or accuracy of the workmanship, but also to give proper directions to any able workman, for making or mending them, when necessary, and even to put to a helping hand himself; I shall therefore here offer some observations and directions for that purpose, immediately regarding those only that are recommended in this book, but which, if well understood, may contribute something towards the construction of other instruments, that are intended for the same use.

I shall suppose the instruments for measuring lines to be well enough explained in the beginning of the First Part, and shall only observe concerning the Chain.

I. The links, or straight pieces of wire, should be all of them exactly of the same length; the wire should be soft, that it may bend any way easily, and not be often exposed to the hazard of breaking, which will be unavoidable, if it is hard: the rings should also be all equal, but the wire of them hard, as well as the hooks

ends of the links to which they are joined, that they may not easily draw out and be lost upon the ground: all the wire should be of the same thickness, about $\frac{1}{10}$ of an inch; which I think is sufficient for strength, and it will be the easier carried about, and drawn along.

II. Instead of the large rings, commonly used at each of the ends, and making the first and last link, with a short piece of wire joined to them, I would recommend iron spikes, about 6 inches long, or a little longer, and $\frac{1}{4}$ of an inch thick, hooked to the wire about the middle, and sharpened at one end for sticking into the ground: for these rings, if they are not very hard, will draw out and make the measure of the line uncertain, by a small odds in every chain's length, if you do not bend them again exactly to the same wideness as at first, which is not so easily done; and if they are very hard, they will be very apt to break: but the spikes are liable to none of these inconveniencies.

The graduated instruments, for measuring angles, require a more particular explanation, for the diameter piece, or radius, and the limb or broad arch upon which the dividing lines are drawn and numbered: the index and fulcrum &c. should need very little explanation to any land-measurer: I shall only observe concerning them, how their goodness may be tried.

The index should be considerably broader at the

centre, and of a thickneſs ſufficient to make it ly cloſe, and ſlide eaſily upon the limb, not turning up edge-wiſe, or bending backwards: its central edge, or fiducial line, ſhould be a true ſtraight line, from the very middle of the centre hole: the centre pin ſhould be exactly round, juſt filling the hole of the index: and the ſights ſhould be ſet truly perpendicular with the very middle of the ſlits above the central edge or fiducial line.

The fulcrum or three-legged ſtaff ſhould be of ſtrength ſufficient to bear the weight of the inſtrument mounted upon it; its head, ball and ſocket, ſcrews and legs, ſhould be all exactly fitted together, that it may not ſhake or tremble in the time of uſing the inſtrument in the field.

If the inſtrument itſelf be all one ſolid piece, whether of wood or braſs, or both, it will have a great deal of needleſs weight; this may be prevented by croſs-bars binding the diameter piece and limb together, and giving the whole a ſufficient ſtrength: theſe bars may be fixed in the following manner, for the graphometer.

Let one bar join the middle of the limb and diameter piece; let another be fixed to this, at right angles, and to the limb on each ſide; let the middle of their joining be about $\frac{2}{5}$ of the radius from the centre; and let their faces, or upper ſides, be exactly in the plain of the limb and diameter piece; ſo as the whole face of the inſtrument may be truly level. To theſe bars, a circu-

lar plate of braſs may be nailed, having a ſocket fixed to it, to receive the neck of the ball in the head of the fulcrum, the middle of the plate upon the middle of the joining of the bars.

When the inſtrument is of wood, and the limb to be covered with a braſs face, for the more exact graduation, the upper part of the limb, and of the ends of the diameter piece, muſt be cut down to the thickneſs of the braſs plate, that the whole face of the inſtrument may be in the ſame plain; and the limb need not be ſo thick as the reſt. But the wood muſt be choſen ſuch as will not eaſily caſt or warp; and the ſeveral pieces, of which the limb is made and the reſt, muſt all be joined together in the beſt manner to prevent, as much as poſſible, that accident, which is ſo common to almoſt all kinds of wood.

A graduated inſtrument may be made with a larger or leſs radius, and the graduation may be more or leſs exact. It is needleſs to inquire, and perhaps not eaſy to determine, what improvements have been made this way: but it ſeems to be probable, that thoſe formerly uſed were not ſo exact as the inſtruments uſed within theſe laſt 100 years: tho' ſome of the improvements of this time, I fear, muſt be owned to have been made the backward way, ſuch as ſhortening the radius &c.

The firſt, or primary, graduation of all inſtruments

is by right lines drawn from the centre, dividing the limb into a certain number of equal parts, which number is greater or leſs in proportion to the radius of the inſtrument; the greater this is, the more exact the inſtrument; the larger the radius, therefore, the better, if no inconvenience attends it. This may be conſidered, and at the ſame time we may ſee what degree of exactneſs can be attained by primary diviſions, which probably was the only kind of graduation uſed formerly, and is that ſuppoſed for ſmall inſtruments in the Introduction.

Let us now ſuppoſe a graphometer of 40 inches radius. It is poſſible, upon a braſs limb, to draw lines for the primary diviſions, to every two minutes; they would be at the equal diſtance of $\frac{1}{43}$ of an inch from one another; and the inſtrument might be reckoned tolerably good, were it not for theſe inconveniencies. 1. I would require a conſiderable breadth of a limb, to hav room for the ſeveral arches, at a proper diſtance, an for figures to diſtinguiſh the larger diviſions from th leſſer, and theſe from one another; and thickneſs alſo fo the ſtrength neceſſary for ſuch a bulk; the breadth an thickneſs of the diameter and bars ſhould be alſo pr portional to the breadth and thickneſs of the limb: th would make it a very heavy inſtrument. 2. The gre quantity of wood and metal, ſuppoſing only a braſs fac and not the whole of braſs, with the price of worki

and graduating it, would make it very dear. 3. It would not be a very eaſy matter to make ſo great a number of ſuch ſmall diviſions exact enough to be truſted, ſo that, after all, it would not be a very ſure inſtrument. 4. Its great bulk and weight, with a proportional fulcrum &c. would make it very unwieldy, and almoſt unmanageable in the field, and conſequently uſeleſs. Great exactneſs, therefore, is not attainable by primary diviſions. A certain number of them is abſolutely neceſſary, but ſome other kind of diviſions, muſt alſo be contrived, or little exactneſs expected: theſe may be called Secondary Diviſions.

Graduation by ſecondary diviſions is of two kinds, viz. by Diagonal Lines, and by a Nonius. By either of which, the inſtrument may be made with a radius of 18 inches, or even a little leſs, more exact than the above ſuppoſed one of 40 inches, and liable to none of its inconveniencies, but light, cheap, and eaſily manageable. I ſhall here explain only the ſemicircular graphometer.

I. Let it be required to have a graphometer made of wood, with a braſs face, the radius 18 inches, and every degree divided into minutes by diagonal lines. The proper directions to the workman, who is ſuppoſed to underſtand the buſineſs, may be ſuch as theſe.

Make the diameter piece and bars $1\frac{1}{2}$ inch broad, and $\frac{1}{2}$ thick, the wooden limb $1\frac{1}{2}$ broad, and $\frac{5}{12}$ thick, and the braſs face $\frac{1}{12}$ thick; let the braſs be well po-

liſhed, and the whole face ſmooth and level: and the Frame, as it may be called, is finiſhed.

Draw one arch, from the centre, quite round upon the limb, and near the outſide of it, eleven more within this, all at the diſtance of $\frac{1}{10}$ of an inch from the firſt, and from one another; then two more cloſe to the inner edge, and to one another.

Divide the firſt arch into 18 equal parts, and draw the dividing lines towards the centre, from the points of diviſion on the firſt or outermoſt arch to the 13th; theſe lines mark out every 10 degrees: divide the ſpaces betwixt them on the firſt arch, each into two equal parts, draw the dividing lines as before, but only half-way from the 12th to the 13th arch; theſe lines mark out every 5 degrees: divide each of theſe ſpaces as before, into 5 equal parts, the dividing lines reaching from the 1ſt to the 12th arch; theſe lines mark out every ſingle degree: and thus you will have all the lines drawn that are neceſſary for the Primary Diviſions. Now for the Secondary or Diagonal Diviſions.

Divide the firſt degree into 6 equal parts, marking the points of diviſion on the 1ſt and 11th arches; theſe points are for every 10 minutes: draw a diagonal from the beginning of the degree, on the 11th arch, to the firſt point on the 1ſt arch; from the firſt point on the 11th to the ſecond on the 1ſt; from the ſecond on the

11[th] to the third on the 1[st] &c. draw a ſhort line from the third point on the 11[th], towards the centre, and half-way to the 12[th] arch; this line marks out the half degree, or 30 minutes. Do the ſame with every degree all round. The interſection of the firſt diagonal, and 10[th] arch, marks out 1 minute; that of the firſt diagonal, and 9[th] arch, 2 minutes; of the ſame diagonal, and 8[th] arch, 3 minutes &c. Set the figures 10, 20, 30 &c. upon, or acroſs the longeſt dividing lines, between the two wideſt arches, making 170 the laſt. Now the graduation is finiſhed, and the inſtrument ready for mounting and uſing.

That every minute may be eaſily diſtinguiſhed, by the above graduation, will not be doubted by any who have ſeen a plain ſcale, ſuch as on the common gunters: the diviſions are the ſame as on the half-inch diagonal ſcale, and the lines may be drawn much finer and truer on braſs, than on wood.

If the radius be much leſs than 18 inches, it will be much more difficult to diſtinguiſh the minutes, and if under 11 inches almoſt impoſſible: $\frac{1}{300}$ of an inch may be diſtinguiſhed by the naked eye, or with the help of a ſmall magnifying glaſs; a radius of 11$\frac{1}{2}$ inches will give that diviſion by diagonals; but I am afraid a ſmaller part cannot be diſtinguiſhed; not to mention the difficulty of making ſuch ſmall diviſions perfectly equal, which ſtill increaſes, as the radius is ſhortened.

II. Let a graphometer be made of 15 inches radius, divided into minutes by a Nonius. The breadth and thickneſs of diameter, limb and bars, may be the ſame as the laſt; and alſo the braſs face to be put on in the ſame manner; but let it be $\frac{1}{8}$ of an inch thick; and let the middle half-inch of its breadth be hollowed or grooved to one half of its thickneſs, the ſides of this groove perpendicular, and the bottom all equal and ſmooth, and continued thro' the diameter piece at the end where the graduation is to begin: thus you will have two limbs, one on each ſide of the groove.

Draw two arches, from the centre, cloſe to one another, and to the outſide of the outward limb, and two more, the ſame way, on the inſide of the innermoſt: theſe may be called bounding arches. Draw two more arches on each limb, $\frac{1}{10}$ of an inch from the edge of the groove, and from one another.

Draw the lines for the primary diviſions on each limb, thoſe for 10's. reaching from the edge of the groove to the neareſt bounding arch, for 5's. within $\frac{1}{10}$ of it, and for the degrees from the ſame edge to the ſecond arch from it. Divide every degree into 4 equal parts, the dividing lines, on each limb, reaching from the edge of the groove to the neareſt arch. Make 4 degrees more on the limbs produced. This is all the graduation of the limbs. Set the figures 10, 20 &c. as before.

Let the index be made of braſs, an inch longer, from the centre, than the longeſt radius, with a cock at the end, to make it ſlide cloſe upon the limbs, and to keep it faſt where-ever it is neceſſary. Let it have a ſmall plate of braſs, projecting from the central edge, of the exact breadth of the groove, and made to fill it up cloſe, as the index moves round; its face being well poliſhed, and ſliding in the plain of the two limbs; its length exactly that of 4 degrees in the groove. Divide its face into 15 equal parts, by right lines from the centre quite a-croſs it, diſtinguiſhing the 5th and 10th from the edge of the index. Theſe are the ſecondary diviſions. This plate is the Nonius: and the graduation is finiſhed.

If you move the index from the diameter to the 4th degree, the end of the Nonius will coincide with the diameter, for its length is exactly 4 degrees: move it a little farther, till its firſt dividing line coincides with the neareſt dividing line on both limbs, and the central edge, or beginning of the Nonius, will cut off an angle as much above 4 degrees, as is the difference betwixt one diviſion of it and one of the ſmalleſt diviſions of the limbs, viz. $\frac{4}{15} - \frac{4}{16}$, or $\frac{1}{60}$ of a degree: this angle, therefore, will be 4 degrees and 1 minute. If you move the index a little farther, till the ſecond dividing line of the Nonius coincide with the next dividing line of the limbs, the angle will now be as much above 4 degrees, as is twice

the above difference, that is 4 degrees and 2 minutes. When the edge of the index comes to the first small division of the limbs above 4 degrees, the end, or 15th division of the Nonius will coincide with the first small division of the limbs, and the angle will be $4° 15'$, and so for any other. So that, by adding the degrees and quarters upon the limbs, nearest the central edge of the index, to the minutes pointed out by the dividing line of the Nonius that coincides with any dividing line of the limbs, you will have the measure of the angle cut off by the index.

I have proposed the groove and two limbs, only for the greater exactness; and that the coinciding of three pretty long lines may be observed the more distinctly: but the whole limb may be only 1 inch broad, without a groove; the brass face may be thinner; the inner half-inch only may be graduated, and the Nonius, of ½ inch broad, may be made to slide upon the limb without the graduated part, having its face sloped inwards, that the dividing lines may the better coincide. I have also proposed the index to be made to reach beyond the limb, only that the cock may have a greater force; but it may be made to reach only as far as the shortest radius, and be joined, by two arms, to a plate of brass, without the limb, an inch broad, a right line thro' the middle of which may be the fiducial line produced, its

inner half may be the Nonius, and the cock may be put upon the back of the outer half; and you may have a Nonius on each ſide of the fiducial line, if you pleaſe. In this laſt caſe, the index may be either of wood or braſs; and the limb need not be continued, or produced thro' the diameter.

The rule for proportioning the Nonius to the diviſions of one degree, on the limb, is this: Divide the number of equal parts, into which the degree is to be divided, by the number of diviſions in one degree, on the limb; the quotient is the number of diviſions of the Nonius; and one more is the number of the diviſions of the limb, to which the length of the Nonius muſt be equal.

Thus, if one degree is primarily divided into 5 parts, and a Nonius be required to give ſecondary diviſions to every half minute; divide 120 by 5, the quotient 24 is the number of the diviſions of the Nonius; and 25 diviſions of the limb, or 5 degrees, is the exact length of it: for $\frac{5}{24} - \frac{5}{25} = \frac{125}{600} - \frac{120}{600} = \frac{5}{600} = \frac{1}{120}$ of one degree, or one half minute &c. the ſame may be done for $\frac{1}{2}$, $\frac{1}{4}$ &c. of a degree.

This is, perhaps, the only method by which an inſtrument of 6 inches radius can be amended.

Theſe inſtruments are, either of them, lighter than the common theodolite, and much more exact; but an inſtrument may be made ſtill lighter, and more exact:

this is a ſmaller graphometer, with the addition of a moving radius, or ſliding degree, or half degree; and ſhall be next explained.

III. Let a graphometer be made of 9 inches radius, divided into minutes by a ſliding degree. Make the whole of the inſtrument of good wood, all of the equal thickneſs of $\frac{1}{3}$ of an inch; the diameter piece and bars $1\frac{1}{2}$ inch broad, and the limb 1 inch: there is no need of any braſs, except a round plate for the centre, if you pleaſe. See Plate IX.

Draw two arches on the limb, the firſt $\frac{1}{10}$ from the outſide, and the ſecond may be $\frac{3}{10}$ from it; draw two more cloſe to the inſide, and to one another. Draw the lines for the primary diviſions to every degree: the 10's. from the firſt arch to the third, the 5's. from the firſt, thro' the ſecond, and half-way to the third, and the degrees from the firſt to the ſecond arch. Set the numbering figures at every two degrees; the 10's. may be larger, and the reſt ſmaller.

Let a kind of wooden groove be made, to move about the centre, upon the limb, with the index in it; the length of which, from the centre, may be $21\frac{1}{2}$ inches; and the groove may be made as follows.

Make two pieces of good wood, each 20 inches long, $\frac{1}{2}$ deep, and $\frac{1}{4}$ broad; theſe will be the ſides of the groove: faſten them, at one end, to a thin plate of braſs

and at the other, to a ſquare piece of wood, having a braſs plate ſunk into it, which may be 1 $\frac{1}{4}$ inch long; theſe are two bottoms: the firſt is to go on the centre, may be 3 inches long, having the centre hole in the middle of the firſt 2, part of which may be rounded, and the ſides fixed to the 3^{d}; the breadth of the bottoms, within the ſides, muſt be exactly that of the index at the fartheſt end, and the ſpace of 1 degree, upon the limb, produced upon the ſunk plate: there muſt be another bottom, made to ſlide round the limb, of the ſame thickneſs with it, and 1 $\frac{1}{2}$ inch long, having a braſs cock, to keep the groove ſliding cloſe upon the limb, and to ſtop it where-ever you pleaſe. Let three ſmall bands, or croſs-bars, bind the ſides together, each $\frac{1}{2}$ inch in breadth, and of a thickneſs nearly equal to what the thickneſs of the index wants of the depth of the ſides; the firſt near the end of the thin braſs bottom, the ſecond near the inner ſide of the limb, and the third near the end bottom: theſe will ſtrengthen the groove, and keep the index from riſing up.

Let the index be made of braſs, having the fiducial line in the middle of it, and a ſquare hole cut thro' it, where it moves upon the diviſions of the limb, one half of this hole on each ſide of that line, which muſt be drawn upon the end of it, that goes over the diviſions, cut ſloping down for that purpoſe. Let the far-

theſt end be broader than the reſt; and let a little more than the 21st inch have one half of its breadth cut out, with ſloping edges, that the diviſions on the braſs plate may be more plainly obſerved, and that the fiducial line may become a central edge there. Fit it now on the inſtrument: produce one dividing line by the edge, the back lying cloſe to one ſide; and the next line, by bringing the edge to it.

Divide this produced degree upon the plate, as directed in p. 296 and 297. This is the ſliding degree: and the inſtrument is now ready to be mounted. The ſliding degree may be firſt made and produced for the graduation of the limb, by moving the groove round &c. It may alſo be divided by a Nonius.

If you bring the groove, ſliding on the limb, with the back of the index cloſe to one ſide, till the fiducial line, in the ſquare hole, coincides with the dividing line on the limb, that is neareſt above the meaſure of the angle required, and keeping it faſt there, move the index on the braſs plate, till the central edge coincides with the line that forms the angle; you will have the number of minutes above the next loweſt degree, and the whole meaſure of the angle, as exactly as if the diameter of the inſtrument were 3 feet and an half.

I have propoſed theſe inſtruments almoſt all of wood, eſpecially the laſt, only for lightneſs and cheapneſs; but

you may make them all of braſs, if you pleaſe; neither the weight nor the price will be very great, conſidering their exactneſs, which is the proper value of an inſtrument.

The conſtruction of the common quadrant differs ſo little from that of the graphometer, that it ſhould need but little explanation. The next inſtrument requires it more.

The bow quadrant has only the limb, a chord fixed to each end of it, and one radius fixed to the middle of the chord and limb, and ſerving for one croſs bar; it may have other two bars fixed the ſame way, each $\frac{1}{4}$ of the chord from its middle. The plate for the ſocket &c. may be nailed to the chord and radius, or middle bar, in four places, at an equal diſtance from the middle of the chord. The index may be made the ſame way as the laſt mentioned, if it is to have a ſliding degree: or it may have one like that of other quadrants, if it is to be graduated by any of the other two methods. The radius may be conſiderably broader on each ſide of the chord, and no more bars will be needful, unleſs the inſtrument be very large.

IV. Let a bow quadrant be made, with a radius of 1 foot, divided into half minutes, by a ſliding degree: and let all, excepting the groove, be made of braſs.

The whole instrument may be cast in one piece, and so as it may be finished up to the following dimensions. See Plate IX.

Make the radius, chord, bars and limb, all of the same thickness, viz. $\frac{1}{8}$ of an inch; let the breadth of them all be also the same, viz. 1 inch; only let the central end of the radius be rounded, with 2 inches diameter, or 1 every way from the centre, and let its breadth on each side of the chord be also 2 inches, or 3 inches, and no more bars.

Draw the same arches as on the limb of the last graphometer, but the second arch $\frac{2}{10}$ from the first.

Draw the primary divisions for every half degree; and number the degrees by two's, as before: the dividing lines for the half degrees may be drawn only $\frac{1}{10}$ from the first arch.

Make a groove in all respects the same as the last only 28 inches long from the centre; which is in the same proportion to the radius, as the last, viz. as 3 to 7. Let the length of the index from the centre be 29 inches, and made like the last, only with the central end to coincide with that of the radius.

Produce two dividing lines containing one degree and also the line for that half degree after the same manner, and divide each of these half degrees, by diagonals into 60 equal parts, as before, or you may produ

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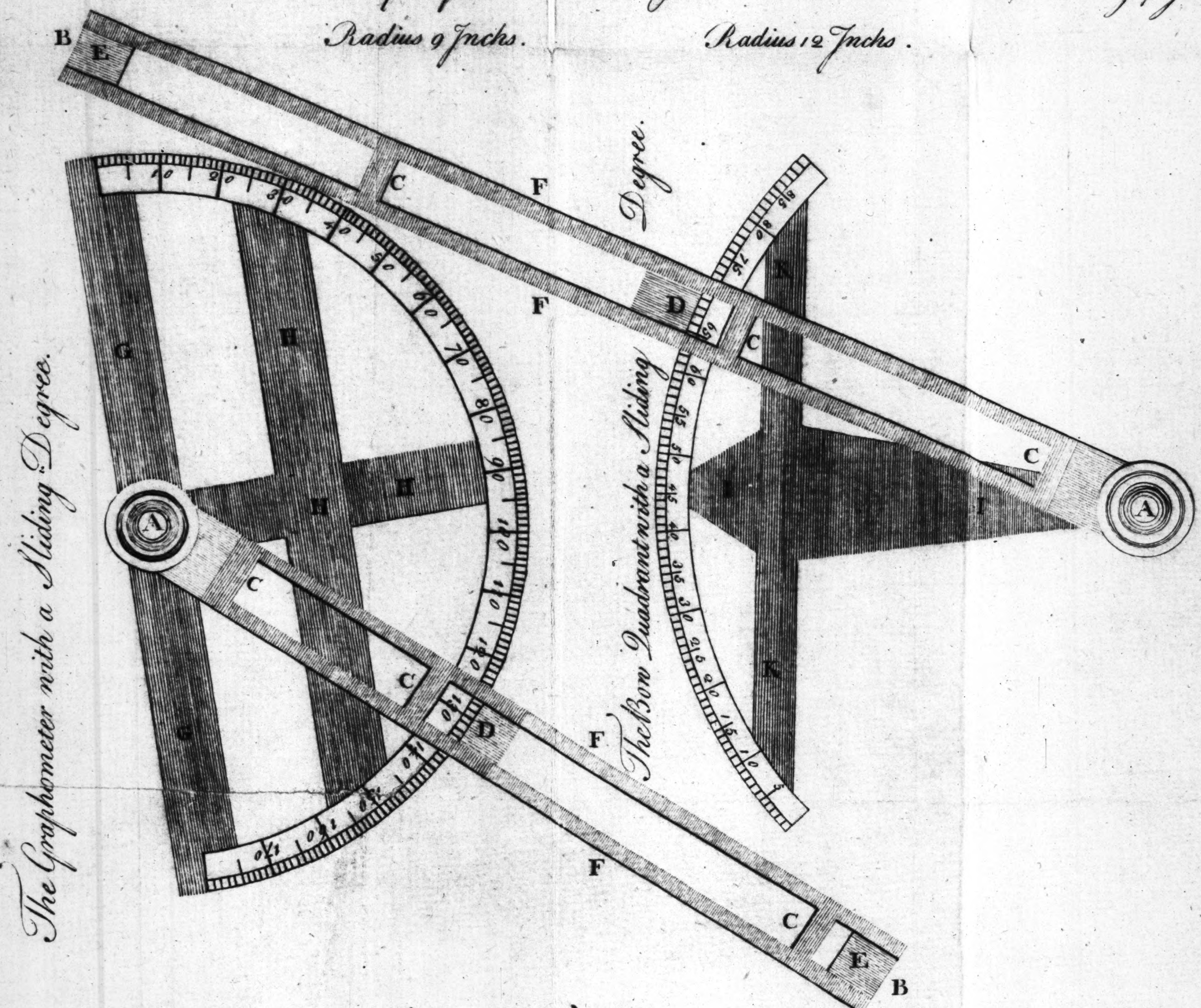
Plate IX. **The Graphometer & Quadrant.** Fronting page 306.



AB The Sliding Grove. C. The Bands. D. The Bottom next the Limb to which the Cock is fixed. E. The Sliding Degree F. The Sides of the Grove A. The Center in a Plate of Brass serving for a Bottom, & rounded at the End. G. The Deameter pice of the Graphomter. H. It's Cross Bras. I. The Radius of the Quadrant. K. Its Chord.

The whole is represented at $\frac{1}{4}$ of the true Deminsions. All except the Grove may be of Brass.

only the half degree, and divide it. In this case the groove may be narrower.

As the proper use of this instrument is to measure an angle of elevation, it will be best to mount it in a perpendicular plain, with the radius to 00° 00′, parallel to the horizon; and to find the measure of the angle, by moving the groove and index.

It may be easily observed, that the bow quadrant, with a sliding degree, may be made more exact, far lighter, and incomparably cheaper, than the best astronomical quadrant yet known. But as this is out of a Land-measurer's way, I shall say no more of it, but conclude this Appendix.

FINIS.

ERRATA.

Page 28. line penult. *read* 10000.
P. 38. l. penult. *read* To Sine of 22° 20′.
P. 76. l. penult. *for* 200. *read* 220.
P. 91. l. 4. *for* : but, *read* . But &c. (in a separate sentence).
P. 112. l. 18. *for* 3,96 *read* 3,94.
P. 146. l. ult. and P. 147. l. 1. *read* 435,635 (and correct the work accordingly).
P. 212. l. ult. *read* 16690 (and correct accordingly).
P. 253. l. 12. *dele* = *before* Dc.
P. 256. *put* C *at the centre of the hexagon.*
P. 257. l. 5. *read* 1273020.

a